SPRING ENGINEERING

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A TEXT-BOOK

FOR ENGINEERS, STUDENTS, AND
DRAUGHTSMEN

—BY—

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JUN. AM. SOC. M. E.

PRICE, $4.00
Delivered in the United States

Published and for sale by
EGBERT R. MORRISON
SHARON, PA., U.S.A.
1915

THE O. S. HUBBELL PRINTING CO
CLEVELAND, OHIO

PREFACE

Springs are among the most ancient of machinery elements; yet because they have lent themselves more or less readily to experimental designing, and because a large amount of work is involved in arriving at their proper dimensions, little investigation has been done along purely scientific lines. Such investigation has been further retarded by a prevalent assumption that a spring is an element of secondary importance. Many mechanisms have failed to give the results and service expected because of a failure to provide springs carefully fitted to the work to be performed.

The following articles have been written largely in answer to the numerous inquiries which followed the publication of our "Spring Tables." It is hoped that they will be of assistance in connection with the better known or more common forms of springs. The work is not intended to cover the field thoroughly and completely, as the subject is one which can scarcely be exhausted.

Such tables as are introduced are taken largely from "Spring Tables," and used here on account of their value in connection with articles along the same lines.

For assistance in developing this book I have to acknowledge my indebtedness: to "Machinery," for permission to reprint articles originally published by them; to The Wm. D. Gibson Co., for many valuable suggestions; and to Mr. Clarence K. Sheers, for assistance in preparing the tabular work.

EGBERT R. MORRISON

December 1, 1915.

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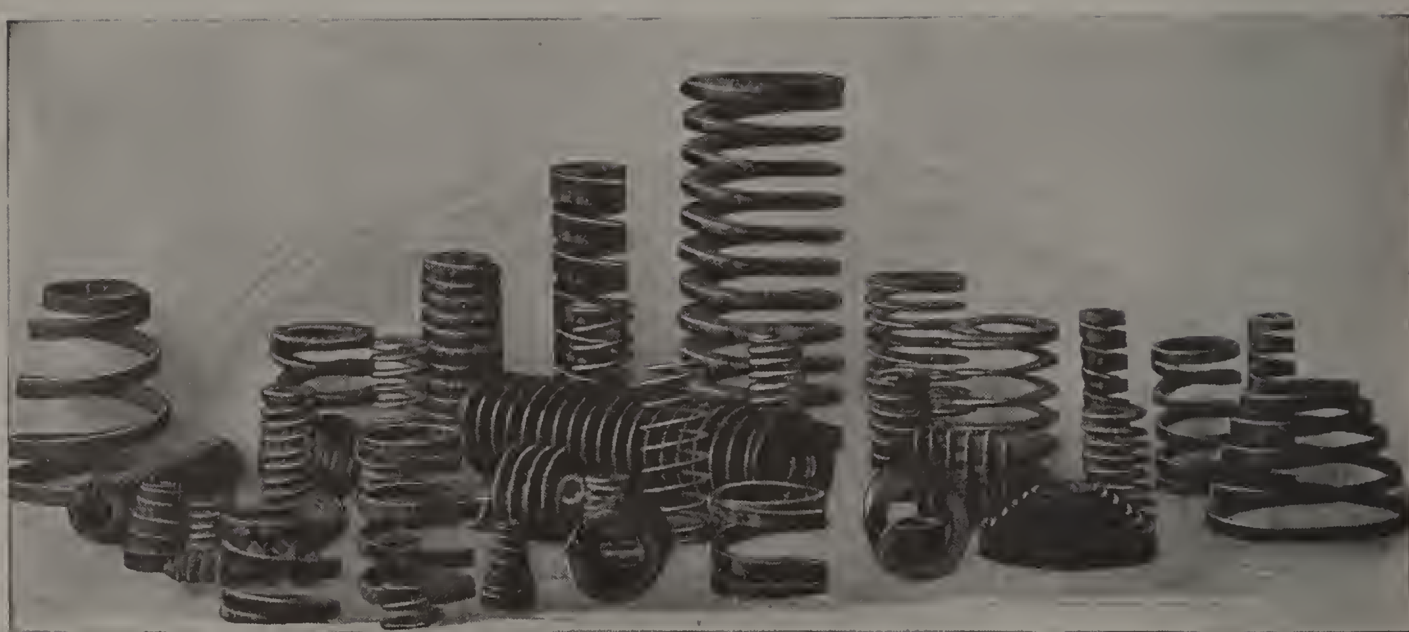


Various Types of Helical Springs

CHAPTER I.

FUNDAMENTAL PRINCIPLES OF SPRING DESIGNING

The fundamental principles of spring designing may be discussed under the headings:

1. Elasticity
2. Physical characteristics
3. Properties of shape
4. Bending formulas
5. Torsion formulas
6. Classification of springs
7. Adaptation of type to work
8. Workmanship

Elasticity

If we bend or twist a hickory stick we find that it opposes our action, and that, if we have not bent or twisted it so far as to destroy its original nature, it will, whether bent or twisted, return to its original shape. All other materials also possess this quality to a greater or less extent. It has been noticed that both the material and shape of the stick have much to do with the extent to which same can be safely distorted, this quality of elasticity, as it is called, depending upon these two properties. Engineers have, therefore, by testing similar shapes made of different materials, and different shapes made of the same material, been able to come to an understanding of the physical properties of matter and the laws of shape in relation to elasticity.

Since the return of a distorted body to its original shape is, in popular language, a "springing" back to original shape, such bodies have come to be known as springs.

Physical Characteristics

A spring is affected by the material of which it is made, because two of the physical characteristics of matter enter into the laws of elasticity. Thus, the safe stress which may be excited within any material depends entirely upon the nature of that material. Again, the extent to which a material can be distorted without resulting in a permanent distortion depends entirely upon the material.

The safe maximum stress is known for different materials solely by experiment and in mathematical discussion is generally represented by S.

The index to the extent of safe distortion is also found by direct experiments, and is called the "modulus of elasticity." Since it is possible to distort materials in different ways there are different "moduli of elasticity." Thus we have,

1. The modulus of elasticity for tension.
2. The modulus of elasticity for compression.
3. The modulus of elasticity for shear.
4. The modulus of elasticity for bending.
5. The modulus of elasticity for twisting.

While springs could, undoubtedly, be designed based upon the action of any of these forces, yet the engineer is concerned practically with only the two classes of springs:

1. Those involving a bending action.
2. Those involving a twisting action.

It is interesting to note that neither of these classes is the result of a simple stress. Bending is a combination of tension and compression. In its application part of the fibres are in tension, part in compression, and between the two there is a neutral line known as "the neutral axis"—neutral to both tension and compression. Twisting may be defined as a special shearing action so applied that the resultant shearing stress in any fibre depends upon its distance from the neutral axis or center line of twist—i.e., the stress diminishes upon approaching the mechanical center, until it is zero or neutral at that center.

It is quite possible, however, that springs based upon the action of direct tension, compression, and shear will in the future demand more thorough investigation in connection with the use of materials like rubber.

The modulus of elasticity for bending is not usually distinguishable from that of tension, so that the effect of the compression present is overlooked. More careful investigation may in the future be very profitable in establishing for different materials the correct modulus of elasticity for bending. The modulus of elasticity both for tension and bending is now generally expressed by E.

The "modulus of elasticity" is a term used to express the ratio between the applied load and the resultant distortion. Thus

$$E = \frac{\text{Force of tension}}{\text{Resultant extension per unit of length}}$$

By custom, unless otherwise stated, the use of the term "modulus of elasticity" relates to the modulus for tension, or E.

In speaking of the other moduli it is usual to say *"for compression," "for torsion,"* etc. Thus, the modulus of elasticity for torsion is usually expressed by G.

The modulus of the torsional elasticity is affected by the fact that the maximum stress acts in an oblique plane and is 25 per cent greater than the stress in the normal plane at any point. Therefore, the maximum value of S must be taken as only $\frac{4}{5}$ of a safe, true maximum stress. In the same way the modulus of elasticity for torsion is not $\frac{1}{2}$ E but $\frac{1}{2}$ ($\frac{4}{5}$ E) = $\frac{2}{5}$ E = G.

Values of E for Various Spring Materials

Steel, cast, spring tempered, high carbon	40,000,000
Steel, P. R. R. analysis spring, tempered	30,000,000
Steel, machinery	30,000,000
Steel, soft	28,000,000
Wrought iron	25,000,000
Platinum	24,000,000
Copper wire	18,500,000
Cast iron	18,000,000
Copper, hammered	15,600,000
Copper, cast	15,000,000
Brass, wire	14,200,000
Bronze, phosphor	14,000,000
Gold	11,500,000
Bronze, gun metal	10,000,000
Aluminum	9,000,000
Lead	2,500,000

The strength of materials is greatly increased by the process of drawing, for which reason the strength of wire exceeds the strength of other forms such as rods, sheets, and castings. The tables given are for direct tension. What might be called the torsional stresses are $\frac{4}{5}$ of these true stresses.

A large manufacturer of brass wire states that brass has a tendency towards brittleness and should, therefore, not be used for severe service. Two-and-one brass is largely used, which, while not so strong as common brass, will withstand more severe service. The maximum toughness for brass is reached in the proportions of 72 per cent copper and 28 per cent zinc.

Gun metal makes a stronger spring but not quite as long lived a spring as phosphor bronze, which is used very largely where repeated action is taking place, as in electrical fixtures.

Brass and bronze wires are liable to become more or less brittle after long storage, so that springs should be made up as soon as possible, the use of the spring seeming to break up the continuity of strains and tendency to crystallization.

Ultimate Resistance to Tension of Bars, Sheets, Castings
Pounds Per Square Inch

Steel	45,000 to 120,000
Steel, aluminum 2⅕ per cent	70,000
Steel, copper 35 per cent	60,000
Steel, nickel 3¼ per cent	86,000
Iron cast	13,400 to 29,000
Iron, wrought	55,000
Copper, cast	19,000
Copper, sheets	30,000
Copper, bolts	36,000
Brass, cast	18,000
Aluminum Bronze	
10 per cent Al. 90 per cent Cu	85,000
1¼ per cent Al. 98¾ per cent Cu	28,000
Aluminum	
Castings	12,000 to 14,000
Sheet	24,000 to 40,000
Bars	28,000 to 40,000
Bronze, gun metal	36,000
Lead, sheet	3,300

Ultimate Resistance of Tension of Wire
Pounds Per Square Inch

Steel, cast, crucible	224,000
Steel, Bessemer	86,600
Steel, high carbon	179,200
Steel, mild O. H.	134,000
Iron, black or annealed	56,000
Iron, bright hard drawn	78,400
Copper, unannealed	60,000
Brass	49,000 to 90,000
Brass, spring tempered	90,000 to 100,000
Bronze, phosphor	130,000 to 140,000
Bronze, gun metal	140,000 to 150,000
Aluminum	25,000 to 55,000

Properties of Shape

The question of shape enters also into spring designing because the shape of a bar decides two things which are controlling factors even as much as the two physical properties of the material. Thus, as we had under physical properties, S and E (or G) so we have under laws of shape c and I (or J).

Upon shape depends the distance of the remotest fibre from the neutral axis. This distance is usually denoted by c and the remotest fibre is the one which by the leverage action of bending or twisting, will be most highly stressed.

Upon shape depends also the distribution, about the neutral axis, of the elementary areas in any cross section, and upon this distribution depends I and J, the rectangular and polar moments of inertia. The load which may be safely applied to distort without permanent distortion, varies directly as the value of these moments of inertia.

The rectangular moment of inertia, I, has to do with bending. The polar moment, G, has to do with twisting.

Fundamental Bending Formulas—Load

When a bar is subject to a bending action there is present an acting moment tending to produce this bending moment and a resisting moment which holds the acting moment in equilibrium.

The acting moment is due to the force P acting at the end of the leverage l and is therefore expressed by Pl.

The resisting moment is expressed by the product of stress times rectangular moment of inertia divided by the distance from the neutral axis, or $\dfrac{SI}{c}$

Then,

Twisting moment = Resisting moment

$$Pl = \frac{S\,I}{c}$$

$$P = \frac{S\,I}{c\,l}$$

Or, since $I = \dfrac{b\,h^3}{12}$ and $c = \dfrac{h}{2}$ for rectangular shapes,

$$P = \frac{S\,b\,h^2}{6}, \text{ the fundamental formula for load.}$$

Fundamental Bending Formula—Deflection

The general equation of the elastic curve applicable to all beams whatever be their shapes, loads, or number of spans is

$$\frac{d^2 y}{d x^2} = \frac{M}{E I}$$

And since $M = P x$

And $$I = \frac{b d^3}{12}$$

$$\frac{d^2 y}{d x^2} = \frac{12 P x}{E b d^3} \text{ for uniform sections.} \qquad \text{(A)}$$

In a cantilever of uniform strength and depth

$$b = \frac{6 P l}{S d^2}$$

$$\therefore \frac{d^2 y}{d x^2} = \frac{2 S}{E d}, \text{ for uniform strength and depth.} \qquad \text{(B)}$$

The double integration of these two equations (A and B) results in the expressions for the elastic curves, and the substitution of $x = l$ gives the maximum deflections, thus:

$$f = \frac{4 P l^3}{E b d^3}, \text{ uniform sections.}$$

$$f = \frac{6 P l^3}{E b d^3}, \text{ uniform strength and depth.}$$

Which are the fundamental formulas for deflection.

Torsion Action of Helical Springs

The same load applied through a radius R, equivalent to the mean radius of helix, will, when applied to a simple torsion spring, produce very nearly the same movement of the load, or the same deflection. As stated by Mr. Oberlin Smith, after making over 200 experiments:

"to ascertain the distance which one end of a proposed spring will move with a given weight attached, it is only necessary to take a straight wire of the same diameter, length, and material, and, fixing one end, twist the other end with the same weight hung from the periphery of a wheel the same size as the mean diameter of the proposed coils—noting, of course, the distance moved by the weight."

There is also a bending action in helical springs but so slight that it is neglected in practical considerations.

Fundamental Torsion Formulas—Load

When a bar is in torsion there is present an acting moment tending to produce this torsion and a resisting moment which holds the acting moment in equilibrium.

The acting moment is due to the force P acting at the end of the radius R, and is therefore expressed as PR.

The resisting moment is expressed by the product of stress times polar moment of inertia divided by the distance from the neutral axis, or $\frac{SJ}{c}$.

Then

$$PR = \frac{SJ}{c}$$

And

$$P = \frac{SJ}{cR}, \text{ the fundamental formula for load.}$$

Fundamental Torsion Formulas—Deflection

If a load, P, acts upon a common or simple torsion spring of length, l, to produce a deflection, then the angle of deflection, or tension, may be represented by θ. If next we consider two sections of this bar δx apart, then the angle of torsion, or relative angular displacement will be

$$d = \frac{PR}{JG}\,\delta x$$

Then by integration the whole angle is

$$\theta = \frac{PRl}{JG}$$

and, since $P = \frac{SJ}{cR}$

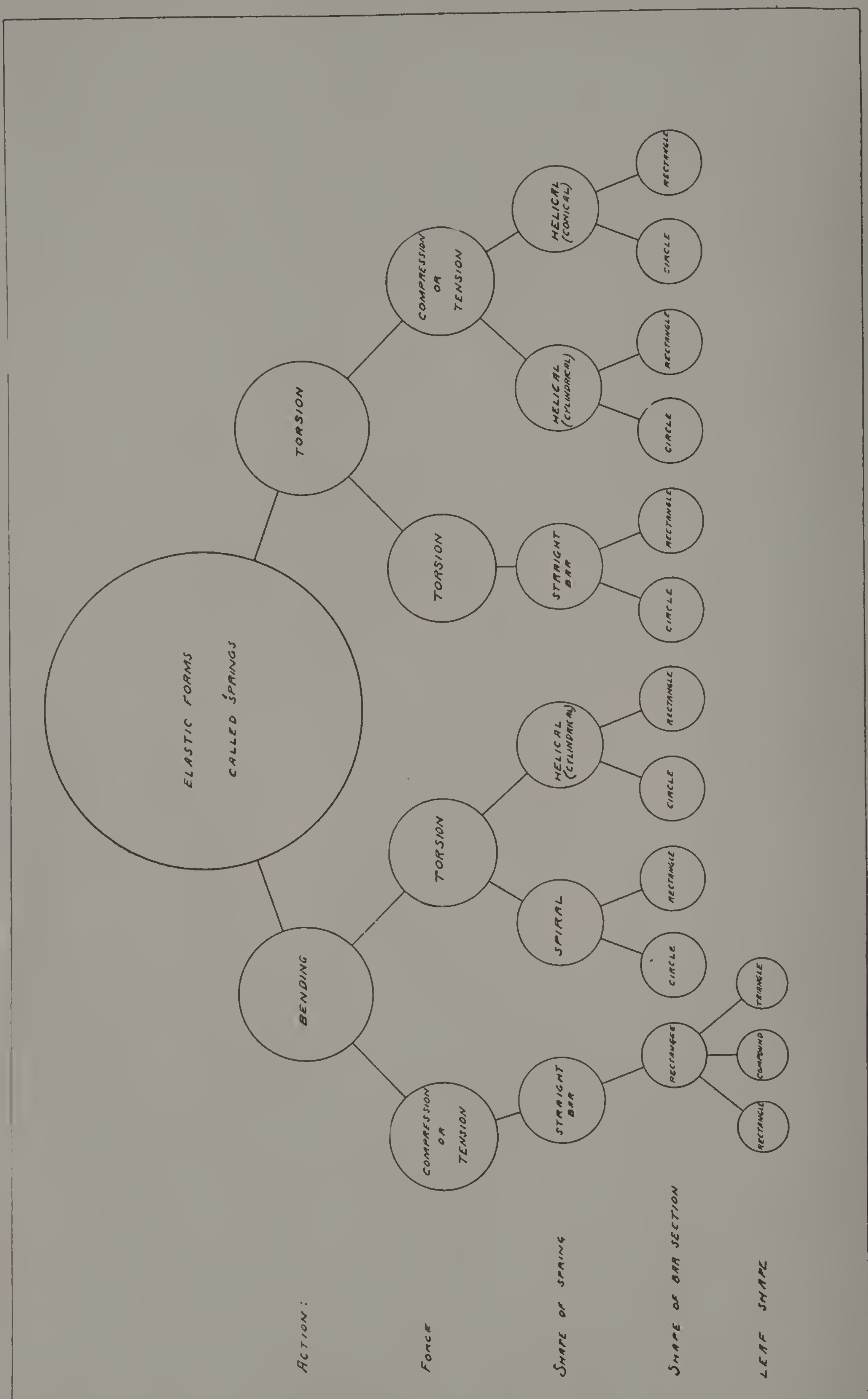
we have, by substitution,

$$\theta = \frac{Sl}{cG}$$

Then since the deflection, f, equals the product of the radius times the angle of deflection, or $f = R\,\theta$ we have $f = \frac{SRl}{cG}$, the fundamental formula of deflection.

Classification of Springs

We are now ready to survey the entire field of springs, a diagram of which shows all the different forms as being related and built upon the fundamental forms.

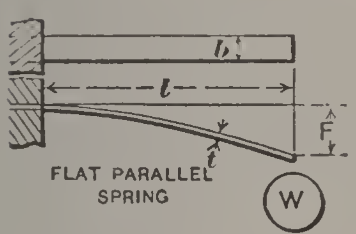


The diagram herewith shows the field of commonly used springs. It can clearly be seen that more complex forms are but combinations

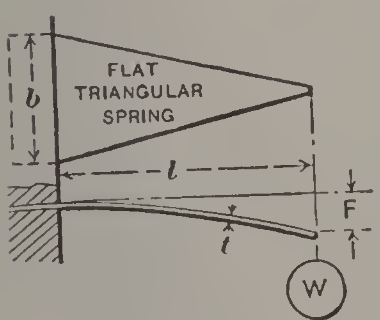
or variations of those in the diagram. As known commercially the more common forms are:

1. Simple rectangular spring (constant section cantilever.)
2. Simple triangular spring (constant strength cantilever).
3. Leaf spring (combinations of the above).
4. Ordinary spiral spring (clocks, etc.).
5. Helical rectangular-bar torsion spring.
6. Helical round-bar torsion spring.
7. Simple rectangular-bar torsion spring.
8. Simple round-bar torsion spring.
9. Helical rectangular-bar compression spring.
10. Helical round-bar compression spring (common).
11. Conical rectangular-bar compression spring (volute).
12. Conical round-bar compression springs (valve spring).
13. Spiral spring, made of round bar (seldom used).

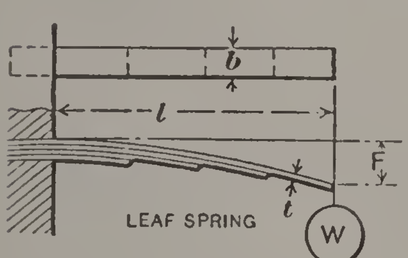
The range of springs may be understood when one considers the heavy elliptics of railroad work weighing a quarter ton or more and the smallest of wire springs which require 38,000 to make one pound —19,000,000 to equal the weight of the larger spring.



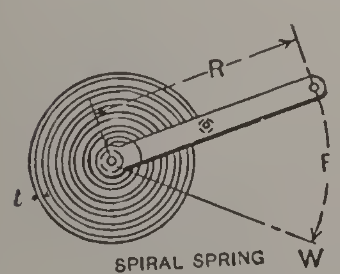
No. 1. (Uniform section cantilever.)



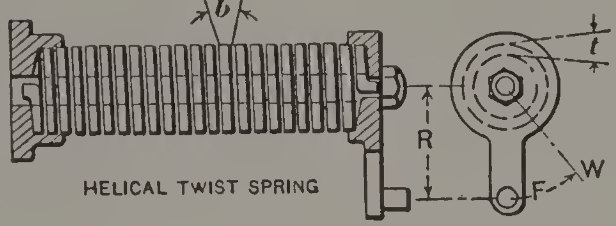
No. 2. (Uniform strength and depth cantilever.)



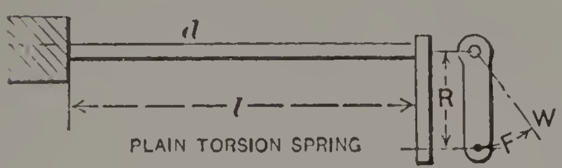
No. 3. (Combination of above.)



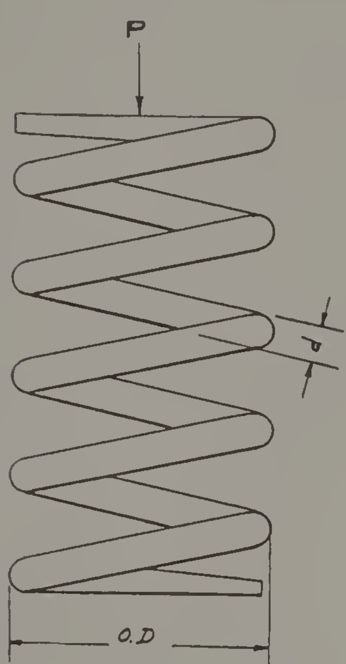
No. 4 or No. 13. (Rectangular or elliptical bars.)



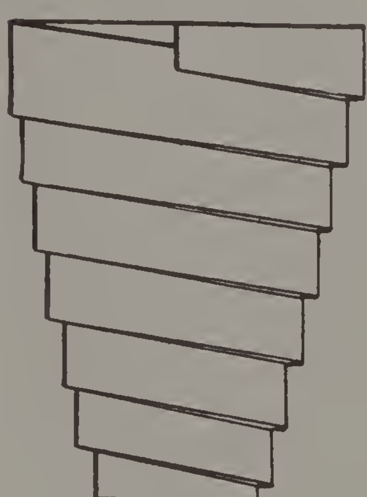
No. 5 or No. 6. (Rectangular or elliptical bars.)



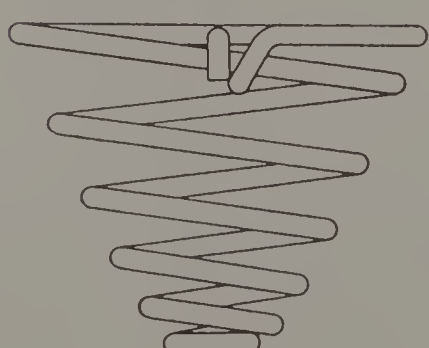
No. 7 or No. 8. (Rectangular or elliptical bars.)



No. 9 or No. 10. (Rectangular or elliptical bars.)



No. 11. (Volute.)



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CHAPTER II.

ELLIPTIC SPRINGS

It is doubtful if scientific calculations ever entered into the design of the original forms of such springs as are used under ordinary road carriages. Satisfactory as they are, they are not engineering results, but accepted standards born long ago of the cut-and-try methods of the blacksmith shop. Their manufacture belongs to such arts as are taught by father to son, or acquired through years of experience, during which have been gathered the "tricks of the trade." The manufacturer of this class of springs does not attempt to arrive at results by mathematics. He has learned as a part of his trade that certain styles of carriages should have certain springs.

Sufficient time did not exist during the development of railroad cars for a gradual development of definite types of springs for various types of cars. It devolved, therefore, upon the engineer to design these springs; but as soon as the spring maker found that the 70,000-, 80,000-, and 100,000-pound capacity car each had its own peculiar set of springs, and that any car could be fitted with springs according to its capacity, he adopted the engineer's designs as another class of standards. Railroad cars, while resting on springs whose dimensions were originally scientifically estimated, are now, therefore, suspended largely upon springs belonging to a few fixed classes.

With the advent of the automobile came a carriage traveling fast over uneven country roads, meeting severe usage in inexperienced hands, and demanding the extreme of comfort and safety. The question of springs and spring suspension thus becomes of primary importance, so that in these carriages each particular design requires a specially designed suspension. Automobile springs are fundamentally cantilevers, the same as all leaf springs. This class of springs more readily lends itself to an easy vibration, as well as to a better general design of the machine. It is possible to carry a load on a narrow-leafed elliptic leaf spring where there would not be room for a helical spring. Also, the addition of a leaf to an elliptic leaf spring adds to its capacity without changing its deflection, while the addition of a coil to a helical spring does not change its capacity but adds to its deflection.

Any leaf spring, tightly banded around the middle, should be considered as composed of two cantilevers of length l, where l is one-half the distance from center to center of the end bearings less one-half the width of the band. The length of each cantilever is then expressed (see Fig. 13):

$$l = \frac{c - w}{2}$$

To consider a spring as a simple beam of length c, is to overlook the effect of the band. It is easily demonstrated that variations in the width of the band cause corresponding variations in the strength and deflection of the spring. The elliptic spring, graduated throughout, with but *one* leaf in each section extending from end bearing to end bearing, is fundamentally a cantilever of *uniform strength;* and the formulas applicable are based on the fundamental formulas of that type of cantilever. An elliptic spring with *all* leaves in each section extending from end bearing to end bearing is, on the other hand, a cantilever of *uniform* section, and the formulas for this type of cantilever are then applicable.

The springs used in automobile practice are frequently combinations of these two forms, inasmuch as a considerable portion of the leaves extend the full length from bearing to bearing. It follows that neither of the above formulas will apply, but that the applicable formulas may be derived by combining the fundamental formulas for the

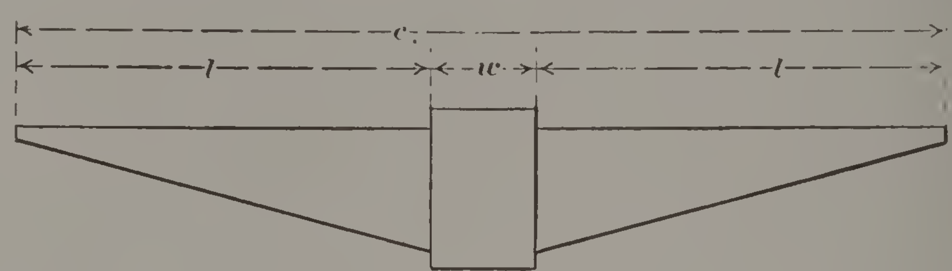


Fig. 13. Diagrammatical Sketch of Graduated Spring, giving Length Notation used in Formulas

two types of cantilevers. The load capacity of a cantilever is not affected by its form, for in either case:

$$P = \frac{S\,b\,h^2}{6\,l}$$

in which $P =$ load,
$S =$ allowable stress,
$b =$ width of beam,
$h =$ thickness of beam,
$l =$ length of cantilever.

In other words, the load capacity is equal for like conditions, such as stress, size of beam, and length of span.

A great difference exists, however, in the deflections under the same load, one being fifty per cent more than the other:

$$f = \frac{4\,P\,l^3}{E\,b\,h^3}, \text{ for uniform section cantilevers,}$$

$$f = \frac{6\,P\,l^3}{E\,b\,h^3}, \text{ for uniform strength cantilevers,*}$$

in which $f =$ deflection, and $E =$ modulus of elasticity.

* The formula given is that for a cantilever of uniform strength, where the height h is uniform, but the width of the section of the cantilever decreases towards the outer end; b is the width at the support.

When such a difference as this exists, it is rather remarkable that many engineers calculate the properties of an elliptic spring no matter what the cantilever conditions as though all elliptic springs were subject to the same rules and formulas; but, as a matter of fact, the proportion of back leaves, or the leaves on the longer side of the spring, which commonly extends the full length, ranges from 5 to 50 per cent of the total number of leaves. It is not unusual to see attempts made through actual tests of the springs themselves to find the proper constant with which to modify the uniform strength equations so as to render them applicable to springs composed of uniform section cantilevers in combination with uniform strength cantilevers. The desired modifier, however, is a variable quantity, depending upon the relative size of the fundamental spring elements.

Lack of due consideration of this combination of different cantilevers accounts also for the different and conflicting formulas which

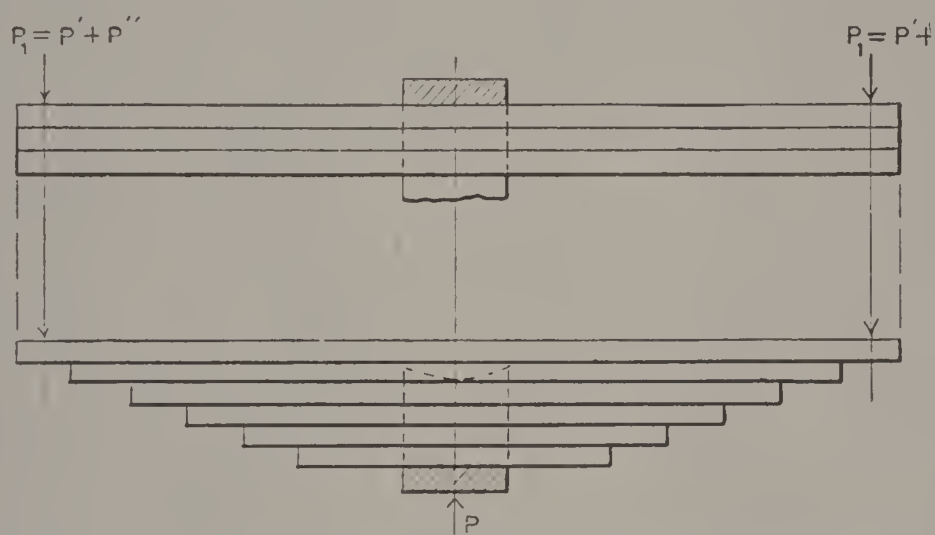


Fig. 14. Showing Division of Spring into Cantilevers of Uniform Section (Upper Portion) and Cantilevers of Uniform Strength (Lower Portion). One of the Full Length Leaves should always be considered as a Part of the Graduated Leaves

various authorities advance. Thus Goodman, in "Mechanics Applied to Engineering"; Reuleaux, in his "Constructor"; and "Des Ingenieurs Taschenbuch" (Hütte), give formulas all of which reduce to uniform strength cantilevers. Molesworth and the Automotor Pocket Book base their formulas on uniform section cantilevers. Henderson, who assumed all semi-elliptic springs to contain one-fourth full length leaves, and made an approximation of the result, was the first to recognize the influence of the combination of cantilevers.

Deduction of General Formulas

For further consideration we will adopt the following notation, discussing only the semi-elliptic spring:

P = total load on spring,

P_1 = portion of load on one end of spring,

P' = portion of load on one end of full-length leaves, or on uniform section cantilever,

P'' = portion of load on one end of graduated leaves, or on uniform strength cantilever,
n = total number of leaves,
n' = number of full-length leaves,
n'' = number of graduated leaves,
$r = \dfrac{n'}{n}$,
S = maximum fiber stress in spring,
S' = maximum fiber stress in full-length leaves,
S'' = maximum fiber stress in graduated leaves,
f = total deflection of banded leaves,
f' = total deflection of full-length leaves if unbanded,
f'' = total deflection of graduated leaves if unbanded,
b = width of leaves,
h = thickness of leaves,
l = length of cantilever,
L = net length of spring, *i. e.*, actual distance between end bearings, less width of band,
x = proper initial space between fundamental cantilevers before banding.

It is but reasonable to assume that the maximum fiber strain should be the same in both fundamental parts, or

$$S' = S''.$$

But

$$S' = \frac{6\,P'\,l}{n'\,b\,h^2},$$

$$S'' = \frac{6\,P''\,l}{n''\,b\,h^2}$$

Hence,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

In a well-designed spring there should be, at full load, a division of the work proportional to the respective number of leaves in the two fundamental parts. The fundamental formulas of the two cantilevers have shown, however, that such proportional loads would produce different deflections in their respective carriers. This difference in deflection would cause a separation of the two portions of the spring were they initially together and unbanded. Were they initially together and banded the result would be internal stress under load which would mean that a division of the load proportional to the respective number of leaves in the two fundamental parts could not exist.

It is evident that by placing a space between the two fundamental parts when unloaded and unbanded, equal to the difference between the two deflections, there will result no space between the two fundamental parts at full load; and hence if banded in this position there

will be no internal stress, so that the load on each part will be proportional to the number of leaves in that part. If then the load be removed, it follows that the band alone holds the two portions together and that there must exist a resulting stress upon the band and leaves.

Now

$$f' = \frac{4\,P'\,l^3}{E\,n'\,b\,h^3} \tag{1}$$

and

$$f'' = \frac{6\,P''\,l^3}{E\,n''\,b\,h^3} \tag{2}$$

But, as shown,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

or

$$P' = \frac{n'\,P''}{n''}$$

Hence $f' = \dfrac{4\,P''\,l^3}{E\,n''\,b\,h^3}$, as derived by substituting in (1).

Hence,

$$f'' - f' = \frac{2\,P''\,l^3}{E\,n''\,b\,h^3}.$$

Also, since

$$\frac{P'}{n'} = \frac{P''}{n''} = \frac{P_1}{n} = \frac{P}{2\,n},$$

we have

$$f'' - f' = \frac{P\,l^3}{E\,n\,b\,h^3}.$$

Also since

$$l = \frac{L}{2},$$

$$f'' - f' = \frac{P\,L^3}{8\,E\,n\,b\,h^3}.$$

or

$$x = \frac{P\,L^3}{8\,E\,n\,b\,h^3}.$$

This last expression is then a general expression of the proper initial distance between the two fundamental portions before banding, expressed in terms of total load on spring, total number of leaves in spring, and net span of spring. To find the actual working deflection of the entire spring it is only necessary now to ascertain how much either portion is deflected by the process of bending. For this purpose let us adopt the following notation:

P_x = force exerted by band,
f_x' = deflection of full-length leaves caused by band,
f_x'' = deflection of graduated leaves caused by band.

Then,

$$f_x' = \frac{2\,P_x\,l^3}{E\,n'\,b\,h^3} \text{ and } f_x'' = \frac{3\,P_x\,l^3}{E\,n''\,b\,h^3}$$

Hence

$$\frac{P_x\,l^3}{E\,b\,h^3} = \frac{f_x'\,n'}{2} = \frac{f_x''\,n''}{3}$$

or

$$f_x' = \frac{2}{3}\left(\frac{1-r}{r}\right)f_x''$$

But

$$f_x' + f_x'' = \frac{P\,l^3}{E\,n\,b\,h^3}$$

Hence

$$f_x'' + \frac{2}{3}\left(\frac{1-r}{r}\right)f_x'' = \frac{P\,l^3}{E\,n\,b\,h^3}$$

$$f_x'' = \left(\frac{3r}{2+r}\right)\frac{P\,l^3}{E\,n\,b\,h^3}$$

But

$$f_x'' = \frac{3\,P_x\,l^3}{E\,n''\,b\,h^3}$$

Hence

$$\frac{3\,P_x\,l^3}{E\,n''\,b\,h^3} = \left(\frac{3\,r}{2+r}\right)\frac{P\,l^3}{E\,n\,b\,h^3}$$

or

$$\frac{3\,P_x\,l^3}{E\,(1-r)\,n\,b\,h^3} = \left(\frac{3\,r}{2+r}\right)\frac{P\,l^3}{E\,n\,b\,h^3}$$

or

$$P_x = \left(\frac{r\,(1-r)}{2+r}\right)P$$

The expression inside the bracket in the above equation becomes zero for either extreme value of r, as would be expected, the extreme values of r being unity and zero. The formula gives the force exerted by the band, *i. e.*, the load upon the band.

The total deflection of the graduated leaves, as already developed, is,

$$f'' = \frac{3\,P\,l^3}{E\,n\,b\,h^3}$$

The deflection of the graduated leaves, caused by the band, is

$$f_x'' = \left(\frac{3\,r}{2+r}\right)\frac{P\,l^3}{E\,n\,b\,h^3}$$

The difference is, therefore, the deflection left in the graduated leaves after banding, or the general formula sought for the deflection of such a spring:

$$f'' - f_x'' = \left\{3 - \left(\frac{3\,r}{2+r}\right)\right\}\frac{P\,l^3}{E\,n\,b\,h^3}$$

or,

$$f = \left(\frac{6}{2+r}\right)\frac{P\,l^3}{E\,n\,b\,h^3}$$

or, since $l = \dfrac{L}{2}$ and

$$P = 2\,P_1 = 2\left(\frac{S\,n\,b\,h^2}{6\,l}\right)$$

$$f = \left(\frac{6}{2+r}\right)\left(\frac{2\,S\,n\,b\,h^2}{3\,L}\right)\frac{L^3}{8\,E\,n\,b\,h^3}$$

Hence

$$f = \frac{1}{2\,(2+r)} \times \frac{S\,L^2}{E\,h}$$

This last expression is then a general formula for the deflection of all semi-elliptic springs. If all the leaves are graduated, $r = 0$, and

$$f = 1/4 \times \frac{S\,L^2}{E\,h}.$$

If all the leaves are full length, $r = 1$, and

$$f = 1/6 \times \frac{S\,L^2}{E\,h}.$$

As was to be expected, the spring composed of all graduated leaves has a deflection, according to the above general formula, 50 per cent above that of a spring composed of all full-length leaves. For values of r above zero, the deflection will be found to decrease until r equals unity.

General Remarks

The general formulas given above were first deduced by the writer in the early part of 1905, at which time they were placed before Prof. C. H. Benjamin, then of the Case School of Applied Science, with a view of making extended experiments for the preparation of a thesis. It was the intention to have springs built with initial space as deduced, and compare the actual deflections of such springs with the estimated deflections. Although these experiments were not carried out, they are mentioned because it is believed that when such experiments are made, they will prove valuable. The deduction of the formulas was published for the first time in January, 1910. This deduction was made in connection with certain springs which were

giving very poor service, although designed by the same formulas as other elliptic springs. It was the writer's conclusion that had the springs been built with the proper initial space between the fundamental parts, these springs would not have broken, and that the omission of this space caused an over-stress in the full-length leaves, and an under-stress in the graduated leaves, which caused the overstrained leaves to break, throwing an overload upon the previously under-stressed leaves, which also broke when the stress became excessive. This conclusion seems to explain why springs of this type are frequently found with only the long leaves broken; the remaining leaves, all being of one type, divide the resultant overload evenly so that the over-stress is not so excessive. Perhaps the strongest indication of the correctness of the deduction lies in the well-known fact that the percentage of breakage is always much greater with semi-elliptic springs (of the combination type, usually) than with full elliptic springs. Also, it is generally found upon unbanding these springs that no initial space exists.

Comparison of deflections estimated from the above formula, with actual deflections, has in some cases been quite satisfactory, while in other cases the actual deflections have appeared closer to those estimated by uniform strength formulas. In such cases where the writer has been able to make comparisons, however, the springs had been *made to specified deflections which evidently were estimated by the uniform strength formulas.* Experienced spring makers know that it is quite possible by putting a "pull" in the springs to vary the deflection and load. This trade term, "pull," is itself nothing more nor less than the introduction of an initial space between the leaves before banding.

Calculations of Springs

The calculation of spring properties by formulas is long and tedious. The writer appends, therefore, a table based on a modulus of elasticity of 25,400,000 and a fiber stress under maximum safe load of 80,000 pounds per square inch. Calculations of springs made of materials having other physical properties are made by simple proportion. This table is to be used only when all leaves are fully graduated.

The safe load on one leaf one inch wide is found by dividing the constant given under P_u by the net length. The corresponding deflection is found by multiplying the constant given under f_u, by the square of the net length.

Example: What is the safe load on a semi-elliptic full graduated spring of five leaves if of one-quarter by two inch steel; length between end bearings, thirty-six inches; band or seat, three inches?

Net length $= 36 - 3 = 33$ inches.

$$\text{Load on one leaf one inch wide} = \frac{3333.33}{33} = 101.01 \text{ pounds.}$$

Semi-Elliptic Spring Table

Giving safe load and deflection for 1 inch wide leaves, 1 inch net length.
Used only when all leaves are fully graduated.

Thickness of Leaf	P_u	f_u	Steel	P_u	f_u
$\frac{1}{32}$	52.08	0.02519	$\frac{9}{32}$	4218.75	0.00280
$\frac{1}{16}$	208.33	0.01260	$\frac{5}{16}$	5208.33	0.00252
$\frac{3}{32}$	468.75	0.00840	$\frac{11}{32}$	6302.08	0.00229
$\frac{1}{8}$	833.33	0.00630	$\frac{3}{8}$	7500.00	0.00210
$\frac{5}{32}$	1302.08	0.00504	$\frac{13}{64}$	8802.08	0.00194
$\frac{3}{16}$	1875.00	0.00420	$\frac{7}{16}$	10208.33	0.00180
$\frac{7}{32}$	2552.08	0.00360	$\frac{15}{32}$	11718.75	0.00168
$\frac{1}{4}$	3333.33	0.00315	$\frac{1}{2}$	13333.33	0.00157

Load on one leaf two inches wide $= 2 \times 101.01 = 202.02$ pounds.
Load on five two-inch leaves $= 5 \times 202.02 = 1010.10$ pounds.
Corresponding deflection is:

$$0.00315 \times (33)^2 = 3.43 \text{ inches.}$$

Formulas can easily be deduced making it possible to use the accompanying table for other classes of elliptic springs than those of the semi-elliptic type with all leaves fully graduated.

The formulas for the semi-elliptic spring with all leaves graduated are:

$$P = \frac{2\,S\,n\,b\,h^2}{3\,L} \text{ and } f = \frac{S\,L^2}{4\,E\,h}.$$

To find the values of P_u given in the table, insert $S = 80{,}000$, $n = 1$, $b = 1$, $h =$ the value given in the first column in the table, and $L = 1$. To find the values of f_u, insert in the second formula $S = 80{,}000$, $L = 1$, $E = 25{,}400{,}000$, and $h =$ the value given in the first column in the table.

Now if the values in the table are to be used for other springs, constants can be deduced by which the table values may be multiplied.

For a semi-elliptic spring with a portion of the leaves graduated the load P remains the same as for a spring with all leaves graduated. The formula for the deflection, however, is:

$$f = \frac{1}{2\,(2 + r)} \times \frac{S\,L^2}{E\,h}.$$

The values in the table, therefore, must be multiplied by the quantity $\frac{2}{(2 + r)} \times L^2$ to find the deflection for any given combination full leaf and graduated spring of effective length L.

For a full elliptic spring with all leaves graduated, P still re-

mains the same as for a semi-elliptic spring, but f doubles its value, or:

$$f = \frac{S L^2}{2 E h}.$$

The values in the table, therefore, in this case must be multiplied by $2 L^2$.

For the full elliptic spring with only part of the leaves graduated, the load P remains the same as before, but the deflection is twice that of a semi-elliptic spring:

$$f = \frac{1}{2(2+r)} \times \frac{2 S L^2}{E h} = \frac{S L^2}{(2+r) E h}.$$

In this case, then, the values for the deflection in the table are to be multiplied by $\frac{4}{2+r} \times L^2$.

The flexibility of a spring is the amount of deflection as compared to the load. This may be expressed as so many inches deflection per hundred pounds, or y.

Example: Assume a full-elliptic, fully graduated spring, where

$S = 80{,}000$,
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$n = 4$,
$h = \frac{1}{4}$ inch,
$L = 30$ inches.

Then the safe load equals:

$$P = 4 \times 1\tfrac{3}{4} \times \frac{3333.33}{30} = 778 \text{ pounds.}$$

And the deflection equals:

$$f = 30^2 \times 2 \times 0.00315 = 5.67 \text{ inches.}$$

Then,

$$y = \frac{5.67}{778} \times 100 = 0.73 \text{ inch.}$$

On the other hand, assume that the thickness and number of leaves are unknown. Then we have:

$P = 778$ pounds,
$S = 80{,}000$
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$L = 30$ inches,
$y = 0.73$ inch.

Then,

$$f = \frac{778}{100} \times 0.73 = 5.67 \text{ inches.}$$

But $f = 2 f_u L^2$, where f_u is the constant for deflection in the accompanying table.

Hence,

$$f_u = \frac{f}{2 L^2} = \frac{5.67}{1800} = 0.00315$$

The thickness of steel in the table which corresponds to this value of f_u is one-fourth inch.

The number of leaves is found by using P_u.

Load on one leaf, one inch wide is:

$$\frac{3333.333}{30} = 111.11 \text{ pounds.}$$

Load on one leaf 1¾ inch wide is:

$$111.11 \times 1\tfrac{3}{4} = 194.25.$$

Number of leaves is then,

$$\frac{778}{194.25} = 4.$$

The present calculation makes no allowance for the leaves of a spring varying in thickness. Where such springs are used, the deflection of the different leaves will not be uniform. Hence, in such springs also a suitable initial "pull" should exist, and such springs should be estimated by a general formula based upon a combination of different cantilevers, thus making allowance for different depths of cantilevers. It is much better to use springs composed of but one thickness of leaves, as the combination of different thicknesses adds a complexity scarcely necessary.

Results obtained from fully graduated full elliptic springs would seem to show that the action of the friction between the leaves is not great enough to seriously affect the bending action, in that the formulas give results agreeing very closely with actual conditions.

It should be noted that the above discussion does not take into account the effect of nesting leaves one inside the other. As a matter of strict fact the leaves of a leaf spring, having thickness, are not free to assume the same radius of curvature, *i. e.*, the actual radii must of necessity vary from one leaf to the next by the thickness of the leaf itself. The consideration of this element adds greatly to the complexity of the problem, and comparisons of actual tests with results deduced by the above formulas seems to indicate that on heavy springs at least such consideration would be an unnecessary refinement.

CHAPTER III.

SPIRAL SPRINGS

Under spiral springs come all flat spirals, such as clock springs, watch springs, etc., also those springs called helical torsion springs. The latter are unlike the former in that the radius of the bar winding is constant instead of increasing. This constant radius makes it necessary for the bar to assume the helical form. The action upon the bar in both cases, however, is one of bending instead of torsion.

The formulas generally given are those of Realeaux, but it should be noted that these formulas are based on the supposition that l is the length of the spring bar and that R represents both the radius of application of the load and the radius of curvature. That such is the assumption is shown in the derivation of these formulas which follows. In order to arrive at the first expression for deflection we have the following:

P is the load.

$\dfrac{D}{2}$ is the leverage or radius.

$$\therefore \left(\frac{D}{2}\right)\left(2\,\pi\,n\right) = \text{linear movement or deflection.}$$

$$\therefore \quad P\left(\frac{D}{2}\right)\left(2\,\pi\,n\right) = \text{Energy expended by bending mo-}$$

ment in twisting spring through n turns.

$$\text{But } P\left(\frac{D}{2}\right) = M. \qquad \text{And } M = \frac{E\,I}{R}$$

Also $2\,\pi\,n = \theta$, the total angular movement

$$\therefore \quad P\left(\frac{D}{2}\right)\left(2\,\pi\,n\right) = \left(\frac{E\,I}{R}\right)\left(\theta\right)$$

$$\therefore \quad \theta = \frac{P\,D\,R\,(2\,\pi\,n)}{2\,E\,I} = \frac{P\,D\,l}{2\,E\,I} = \frac{M\,l}{E\,I}$$

$$f = \theta\,R = \frac{M\,l\,R}{E\,I} = \frac{P\,R^2\,l}{E\,I}, \text{ the general expression for deflection.}$$

Here l is not, as assumed, the length of the bar which forms the spring, for $\pi\,D\,n$ is simply the linear expression for the deflection, which becomes equal to the length of the spring *only* if the spring becomes a straight bar when released. In other words if a tempered straight bar were bent so that the force applied followed the bar and actually moved through the linear distance, $\pi\,D\,n$, then $\pi\,D\,n$ would become the expression for length of bar as well as deflection of load.

Also, if in one case R is the radius of the bending moment, and in another the radius of curvature, then this assumption means also that the spring is a straight untempered bar when released of all load; otherwise R cannot be the radius of curvature.

The formulas of Reuleaux, the derivation of which follows, are therefore not applicable to a spring which is of spiral or helical shape when not under stress. The attempts to fit these formulas to such springs has led to much confusion. The writer knows of no formulas which have yet been developed which are intended to apply to unstressed spiral or helical shapes used under torsional loads. The derivation of such formulas and comparison of same with practical results should be a field for interesting research.

The general expression for load is the simple fundamental formula

$$M = \frac{S\,I}{c}.$$

From the above we proceed as follows:

Circular Bars—Deflection of Spiral Springs

As before, $f = \dfrac{P\,R^2\,l}{E\,I}$

But $I = \dfrac{\pi\,d^4}{64}$ $\qquad \therefore\quad f = \dfrac{64\,P\,R^2\,l}{\pi\,d^4\,E}$

Circular Bars—Load of Spiral Springs

As before, $M = \dfrac{S\,I}{c}$ $\qquad$ And $P\,R = \dfrac{S\,I}{c}$ $\qquad$ And $P = \dfrac{S\,I}{c\,R}$

But, $c = \dfrac{d}{2}$ $\qquad$ And, $I = \dfrac{\pi\,d^4}{64}$ $\qquad \therefore\quad P = \dfrac{\pi\,d^3\,S}{32\,R}$

Rectangular Bars—Deflection of Spiral Springs

As above, $f = \dfrac{P\,R^2\,l}{E\,I}$

But, $I = \dfrac{b\,h^3}{12}$ $\qquad \therefore\quad f = \dfrac{12\,P\,R^2\,l}{E\,b\,h^3}$

Rectangular Bars—Load of Spiral Springs

As above, $M = \dfrac{S\,I}{c}$ $\qquad \therefore\quad P\,R = \dfrac{S\,I}{c}$ $\qquad$ And $P = \dfrac{S\,I}{c\,R}$

But, $c = \dfrac{h}{2}$ $\qquad$ And, $I = \dfrac{b\,h^3}{12}$ $\qquad \therefore\quad P = \dfrac{S\,b\,h^2}{6\,R}$

CHAPTER IV.

HEAVY HELICAL SPRINGS

A spring is usually specified by three dimensions, although some specifications complete the design by a fourth. The dimensions usually given are the outside diameter, free height, and diameter of bar. The fourth dimension, the solid height, is not generally given, so that the actual design of the spring is really left to the manufacturer. In some cases the number of coils or "rings" is specified, but this should never be done, as a tapered coil may be considered by one as a full coil and by another as a partial coil, thus causing confusion.

Investigation of such formulas as are found in the general text-books, hand-books, and books of reference, indicates the need of more direct formulas to facilitate the design of springs. It is the writer's intention to present the derivation of such formulas with parallel examples, showing the ease of application. For this purpose we adopt the following notation:

d = diameter of bar,
D = mean diameter of coil,
f = total deflection,
h = solid height,
H = free height,
L = blunt length of bar,
W = weight of bar, or spring,
P = capacity of coil,
P_1 = any load less than capacity,
h_1 = height of coil under load P_1,
S = maximum fiber stress,
G = torsional modulus,
w = weight of steel per cubic inch.

Only round bar coils will be considered.

I. Length of Bar When Solid Height is Given

Total number of coils $= \dfrac{L}{\pi D}$.

Total number of coils $= \dfrac{h}{d}$.

Hence,

$$\frac{L}{\pi D} = \frac{h}{d}.$$

$$L = \pi \left(\frac{D}{d}\right) h = 3.1416 \left(\frac{D}{d}\right) h$$

Example: Outside diameter = 4⅜ inches,
Bar = 7/16 inch,
Solid height = 10 inches.

$$L = 3.1416 \times \left(\frac{3\frac{15}{16}}{\frac{7}{16}}\right) \times 10 = 282.74 \text{ inches.}$$

II. Deflection When Solid Height is Given

Fundamentally, as given in most text-books,

$$f = \frac{L\,D\,S}{G\,d}.$$

But,

$$L = \pi \left(\frac{D}{d}\right) h$$

Hence,

$$f = \frac{\pi\,S}{G} \left(\frac{D}{d}\right)^2 h$$

Or, for steel springs,

$$f = 0.019946 \left(\frac{D}{d}\right)^2 h$$

Example: Outside diameter = 4¼ inches,
Diameter of bar = ¾ inch,
Solid height = 10 inches.

$$f = 0.019946 \left(\frac{3\tfrac{1}{2}}{\tfrac{3}{4}}\right)^2 \times 10 = 4.34 \text{ inches.}$$

III. Ratio Between Free and Solid Heights

$$H = h + f$$

$$f = \frac{\pi\,S}{G} \left(\frac{D}{d}\right)^2 h$$

Hence,

$$H = h + \frac{\pi\,S}{G} \left(\frac{D}{d}\right)^2 h$$

$$H = \left[1 + \frac{\pi\,S}{G} \left(\frac{D}{d}\right)^2\right] h$$

Or, for steel springs,

$$H = \left[1 + 0.019946 \left(\frac{D}{d}\right)^2\right] h$$

and

$$h = \frac{H}{1 + 0.019946 \left(\frac{D}{d}\right)^2}$$

Example 1: Outside diameter = 6 inches.
Diameter of bar = 1⅛ inch,
Free height = 13¾ inches.
Find solid height h.

$$h = \frac{13.75}{1 + 0.019946 \left(\frac{4\tfrac{7}{8}}{1\tfrac{1}{8}}\right)^2} = 10 \text{ inches.}$$

Example 2: Outside diameter = 7⅛ inches,
Diameter of bar = 1⅛ inch,
Solid height = 10 inches.
Find free height *H*.

$$H = \left[1 + 0.019946 \left(\frac{6}{1\frac{1}{8}} \right)^2 \right] \times 10 = 15.67 \text{ inches.}$$

IV. Deflection When Only Free Height is Given

$$f = \frac{\pi S}{G} \left(\frac{D}{d} \right)^2 h$$

But

$$h = \frac{H}{1 + \frac{\pi S}{G} \left(\frac{D}{d} \right)^2}$$

Hence,

$$f = \frac{\frac{G}{\pi S} \left(\frac{D}{d} \right)^2 H}{1 + \frac{\pi S}{G} \left(\frac{D}{d} \right)^2}$$

$$f = \frac{H}{1 + \frac{G}{\pi S} \left(\frac{D}{d} \right)^2}$$

Or, for steel springs,

$$f = \frac{H}{1 + 50.1337 \left(\frac{d}{D} \right)^2}$$

Example: Outside diameter = 5½ inches,
Diameter of bar = 1⅜ inch,
Free height = 11¾ inches.

$$f = \frac{11\frac{3}{4}}{1 + 50.1337 \left(\frac{1\frac{3}{8}}{4\frac{1}{8}} \right)^2} = 1\frac{3}{4} \text{ very nearly.}$$

V. Weight When Solid Height is Given

$$\text{Area of cross section} = \frac{\pi d^2}{4}.$$

$$\text{Cubical contents of bar} = \frac{L \pi d^2}{4}.$$

$$\text{Then } W = \frac{L \pi d^2 w}{4}$$

$$\text{But } L = \pi \left(\frac{D}{d}\right) h$$

$$\text{Hence, } W = \frac{\pi^2 w}{4} d D h$$

For steel springs, where one cubic foot of steel weighs 486.6 pounds,

$$W = 0.694\ d D h.$$

Example: Outside diameter = 3¾ inches,
Diameter of bar = 15/16 inch,
Solid height = 10 inches.

$$W = 0.694 \times \frac{15}{16} \times 2\frac{13}{16} \times 10 = 18.3 \text{ pounds.}$$

VI. When Free and Solid Heights are Given to Determine Stress

$$h = \frac{H}{1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2}$$

$$S = \frac{(H - h)\, G}{\pi h} \times \left(\frac{d}{D}\right)^2$$

$$S = \frac{G f}{\pi h} \times \left(\frac{d}{D}\right)^2$$

For steel springs,

$$S = 4{,}010{,}700\ \frac{f}{h}\left(\frac{d}{D}\right)^2$$

Example: Outside diameter = 4½ inches,
Diameter of bar = ½ inch,
Free height = 22¾ inches,
Solid height = 10 inches.

$$S = 4{,}010{,}700 \times \frac{12.75}{10}\left(\frac{0.5}{4}\right)^2 = 80{,}000 \text{ pounds.}$$

VII. When Free and Solid Heights are Given to Determine Capacity

$$P = \frac{S \pi d^3}{8 D}$$

and

$$S = \frac{G f}{\pi h} \left(\frac{d}{D} \right)^2$$

Hence,

$$P = \frac{G f d^5}{8 h D^3}.$$

For steel springs,

$$P = 1{,}575{,}000 \frac{f d^5}{h D^3}$$

Example: Outside diameter = 2⅞ inches,
Diameter of bar = ½ inch,
Free height = 14½ inches,
Solid height = 10 inches.

$$P = 1{,}575{,}000 \times \frac{4.5 \times 0.5^5}{10 \times 2.375^3} = 1653 \text{ pounds.}$$

These last two formulas are very useful in ascertaining the stresses and loads of the separate coils of double and triple coil springs.

VIII. Given Free Height, Diameter of Spring and Bar, and Load Carried at Given Height. To Find Proper Solid Height

$$\frac{P_1}{P} = \frac{f_1}{f}$$

$$H = f + h$$

$$H = f_1 + h_1$$

Hence, $f_1 = f + h - h_1$

Then $P\ (f + h - h_1) = P_1 f$

Hence $h = \dfrac{P_1 f - Pf + Ph_1}{P}$

$$h = \frac{P_1 - P}{\mathrm{P}} \times f + h_1$$

But $f = \dfrac{\pi S}{G} \left(\dfrac{D}{d} \right)^2 h$

Hence,

$$h = \frac{\pi S}{G} \left(\frac{D}{d} \right)^2 \left(\frac{P_1 - P}{P} \right) h + h_1$$

$$h = \frac{h_1}{1 + \frac{\pi S}{G}\left(\frac{P - P_1}{P}\right)\left(\frac{D}{d}\right)^2}$$

For steel springs,

$$h = \frac{h_1}{1 + 0.019946\left(\frac{P - P_1}{P}\right)\left(\frac{D}{d}\right)^2}$$

Example: Outside diameter = 5½ inches,
Diameter of bar = ¾ inch,
Free height = 18 inches.

What solid height is required for carrying 1395 pounds at 14 inches?

$P = 2970$ pounds by formula $P = \frac{S \pi d^3}{8 D}$.

Then,

$$h = \frac{14}{1 + 0.019946\left(\frac{2790 - 1395}{2790}\right)\left(\frac{4\frac{3}{4}}{\frac{3}{4}}\right)^2} = 10 \text{ inches.}$$

IX. To Determine the Quality of the Steel

The value of G is the index to the quality of the steel, and upon this value depend all properties of the spring. By transposing the formulas given in (VII) for stress and load we find a method for ascertaining this value, *i. e.*:

$$G = \pi S \frac{h}{f}\left(\frac{D}{d}\right)^2 = 8 P \frac{h D^3}{f d^5}$$

Example: Outside diameter = 4⅞ inches,
Diameter of bar = 11/16 inch,
Load = 1219 pounds,
Deflection = 3.7 inches,
Solid height = 10 inches.

$$G = 8 \times 1219 \times \frac{10 \times (4\frac{3}{16})^3}{3.7 \times (\frac{11}{16})^5} = 12{,}600{,}000.$$

General Remarks

Concentric coils, as shown in Fig. 18, are made generally of the same free and solid heights. Presuming that such coils are all made of the same quality of steel, the ratio of $\frac{D}{d}$ should be the same throughout, for the formula in (II) clearly shows that this is necessary to obtain equal stresses in all coils.

The formula in (1) shows that when all values of $\frac{D}{d}$ are made the same, the lengths of all bars will be the same before tapering. A

study of all the formulas reveals the fact that the ratio of $\frac{D}{d}$ determines everything; the writer emphasizes the importance and meaning of this ratio by terming it the *spring index.*

The absolutely perfect design of concentric springs is seldom possible where a scale of sixteenths inch for dimensions is used, with the customary one-eighth inch between inside diameter of one spring and outside diameter of the next. As cases of perfect design, however, the following springs are given as examples:

Outer: 5 inches outside diameter, 15/16 inch bar, index 4⅓.

Inner: 3 inches outside diameter, 9/16 inch bar, index 4⅓.

Fig. 15. Typical Heavy Helical Springs

Outer: 2⅝ inches outside diameter, ⅜ inch bar, index 6.
Inner: 1¾ inch outside diameter, ¼ inch bar, index 6.

In concentric coil springs where perfect design is impossible, the coil having the least value of $\frac{D}{d}$ will be stressed the highest, as shown by the formula in (VI); this coil may therefore be called the governing coil, inasmuch as the motion, or deflection, of the spring as a whole depends upon this coil. To estimate the capacity of such concentric coils we have recourse to the formula in (VII), while the formula in (VI) shows the separate stresses. The load which the concentric spring will carry at any height is then found by the fact that all loads are proportional to deflection.

In actual design adjacent coils are wound in opposite directions to prevent binding, as shown in Fig. 18. Instead of using concentric coils, groups of similar coils are sometimes used which are held together by pressed steel or cast spring-plates, as shown in Fig. 16. It is customary to suspend the static load at one-half the deflection.

CHAPTER V.

GROUPED HELICAL SPRINGS

It is the intention to present here, a study of the design of grouped helical springs, developing the subject upon the basis of the relation which exists between the diameter of the bar and the mean diameter of the spring. In the discussion only round bar coils will be considered.

Notation

The following notation will be adopted:

$S =$ stress solid, or maximum stress, usually assumed to be 80,000 pounds per square inch for heavy steel springs;

$G =$ modulus of torsional elasticity, taken at 12,600,000 pounds per square inch for steel springs;

Fig. 16. Groups of Coil Springs held together by Plates at Top and Bottom

$w =$ weight of one cubic inch of the spring material;
$\pi = 3.1416$;
$f =$ total deflection;
$H =$ free height;
$h =$ solid height;
$h_1 =$ any other height;
$P =$ capacity at solid height, or weight necessary to produce complete deflection;
$P_1 =$ load at h_1, or weight necessary to compress to h_1;
$W =$ weight of spring;
$L =$ blunt length of bar, or length before tapering;
$D =$ mean diameter of coil;
$d =$ diameter of round bar;
$r =$ spring index, or $D \div d$.

Definitions

In addition to the above notation, the following definitions will serve to clear the discussion:

Spring: any single coil, combination, or group of coils.

Coil: a spring composed of one bar only.

Turn: a wind or rotation, a part of a coil.

Turns are fundamental elements; coils are composed of winds; and *springs* consist of one or more coils.

The Spring Index

The *deflection* of a helical spring may be expressed as

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h \qquad (1)$$

The *capacity* may be expressed by

$$P = \frac{\pi S}{8}\left(\frac{d}{D}\right) d^2 \qquad (2)$$

The *weight* may be expressed as

$$W = \frac{\pi^2 w D^2 h}{4}\left(\frac{d}{D}\right) \qquad (3)$$

The *length of bar* to form the spring may be expressed

$$L = \pi\left(\frac{D}{d}\right) h \qquad (4)$$

These four standard equations being solved, the length of bar required to make the spring will be known, as well as the spring weight, capacity and deflection. A further inspection of these formulas will show that all properties depend upon the ratio between the diameter of the bar and the mean diameter of the spring. This all-important ratio we have, in the preceding chapter, termed the *spring index*, expressed as

$$r = \frac{D}{d}$$

Fundamental Principle of Grouped Springs

Equation (2) gives one value of $\frac{d}{D}$, while equation (3) affords another. Equating,

$$\frac{8P}{\pi S d^2} = \frac{4W}{\pi^2 w D^2 h} \qquad (5)$$

Whence

$$P = \frac{WS}{2\pi w h}\left(\frac{d}{D}\right)^2 \qquad (6)$$

This is the fundamental principle of grouped spring design and means that when a constant weight of material is uniformly stressed, the resultant capacity varies inversely as the square of the spring index, and that the actual number of coils or dimensions thereof is immaterial for a constant weight and spring index.

To Ascertain the Value of the Spring Index

Having given the desired capacity, free height, solid height and material of a spring, it may further be assumed that the maximum fiber stress and modulus of elasticity are also known. If then, the spring index be ascertained, the ratio of mean diameter of coil to diameter of bar that must be maintained in order to produce the results desired, will thus be given.

Fig. 17. Double and Triple Coil Concentric Springs

The value of the spring index from equation (1) is,

$$r = \frac{D}{d} = \sqrt{\frac{f\,G}{\pi\,S\,h}} \tag{7}$$

which may readily be solved since $f = H - h$, the difference between two known quantities.

Constant Areas, the Basis of Bar Sizes and Dimensions

No matter of how many bars or coils the spring unit may be composed, the sum of the cross-sectional areas of the individual bars is constant. This fact furnishes a basis from which to ascertain the sizes and dimensions of the bars, according to whether there is one or

more coils used. It is important, therefore, to deduce an expression for this constant area. Consider a single coil spring.

The product of equations (1) and (2) is

$$P\,f = \frac{\pi^2 S^2 d\,D\,h}{8\,G}$$

which may be expressed

$$Pf = \frac{S^2}{2\,G}\left(\frac{\pi^2 d\,D\,h}{4}\right) \tag{8}$$

Then

$$\frac{\pi^2 d\,D\,h}{4} = \frac{2\,G\,P\,f}{S^2} \tag{9}$$

Equation (3) may now be written

$$W = \left(\frac{\pi^2 d\,D\,h}{4}\right) w \tag{10}$$

Substituting the value of $\frac{\pi^2 d\,D\,h}{4}$ as given in equation (9)

$$W = \frac{2\,G\,P\,f\,w}{S^2} \tag{11}$$

The total weight divided by the unit weight will give the volume, or

$$V = \frac{W}{w} = \frac{2\,G\,P\,f}{S^2}$$

From equation (4) the length of the bar will always be

$$L = \pi\,r\,h \tag{12}$$

The volume, divided by this constant length, will therefore result in an expression of the constant area, or

$$\frac{V}{L} = A = \frac{2\,G\,P\,f}{\pi\,S^2\,r\,h} \tag{13}$$

Substituting from equation (1) the value of f, placing $\frac{D}{d} = r$, gives

$$A = \frac{2\,P\,r}{S} \tag{14}$$

which is the constant area value sought, and being in known terms may be readily obtained.

This applies equally well for a multi-coil spring, for the weight is uniformly taken up by each of the units of the spring. Therefore the total cross-sectional area is constant.

Determinate Equations for Bar Sizes and Dimensions

In concentric coils, let all the properties of the inner coil be denoted by the subscript 1; of the next coil by the subscript 2; of the next coil by the subscript 3; and so forth. The total sectional area will then always be

$$\frac{\pi d_1^2}{4} + \frac{\pi d_2^2}{4} + \dots\dots\dots + \frac{\pi d_n^2}{4} = \frac{2 P r}{S} \qquad (15)$$

which may be expressed

$$\frac{\pi}{4} (d_1^2 + d_2^2 + d_3^2 + \dots\dots\dots + d_n^2) = \frac{2 P r}{S} \qquad (16)$$

It is possible also from the relation of the diameters of the coils to form as many equations as there are coils less one, so that there

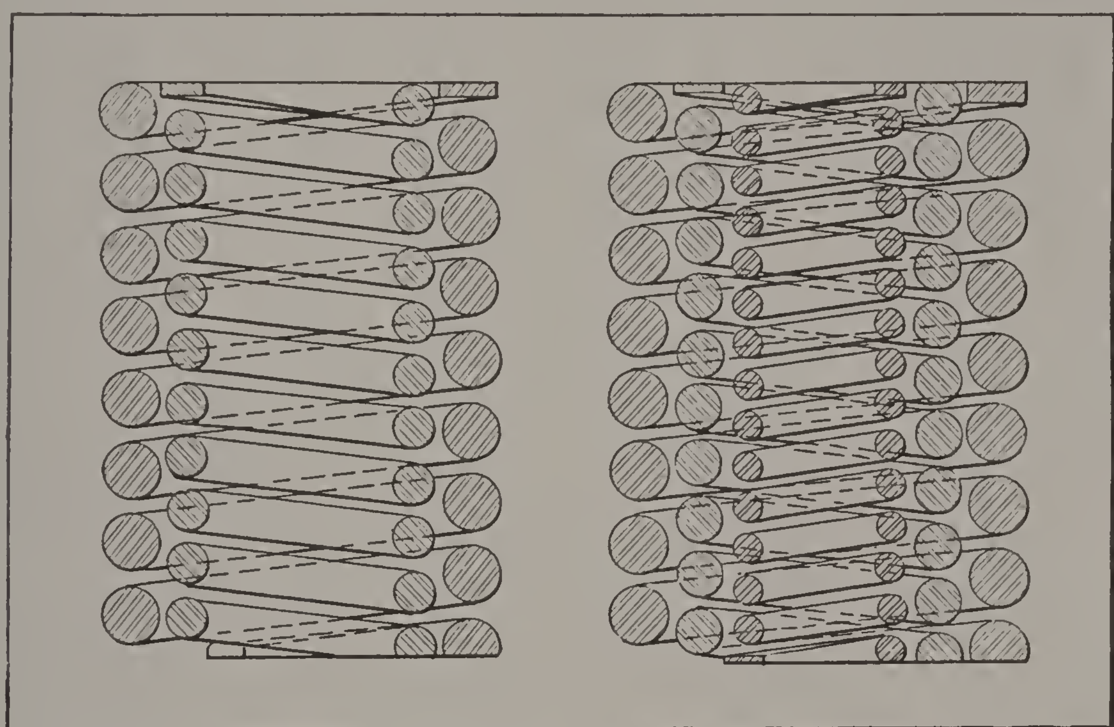


Fig. 18. Double and Triple Coil Concentric Groups, showing Right-hand and Left-hand Coiling, to prevent Binding

may be always found as many equations as there are unknown quantities, or bars, or values of d.

Equations based on the relations of coils are deduced as follows, where D_n' = inside diameter and D_n'' = outside diameter of nth coil.

$$D_n'' = D_n' + d_n = d_n \left(\frac{D_n'}{d_n}\right) + d_n \qquad (17)$$

or

$$D_n'' = (r + 1)\, d_n \qquad (18)$$

In the same way

$$D_n' = (r - 1)\, d_n \qquad (19)$$

Then let the difference between the outside diameter of one coil and the inside diameter of the next be taken as any desirable clearance, c. Or

$$D_n' - D''_{n-1} = c \qquad (20)$$

This gives the series of equations sought, thus:
Between first and second coils,

$$(r-1)\ d_2 = (r+1)\ d_1 + c \tag{21}$$

Between second and third coils,

$$(r-1)\ d_3 = (r+1)\ d_2 + c \tag{22}$$

Between third and fourth coils,

$$(r-1)\ d_4 = (r+1)\ d_3 + c \tag{23}$$

and so forth.

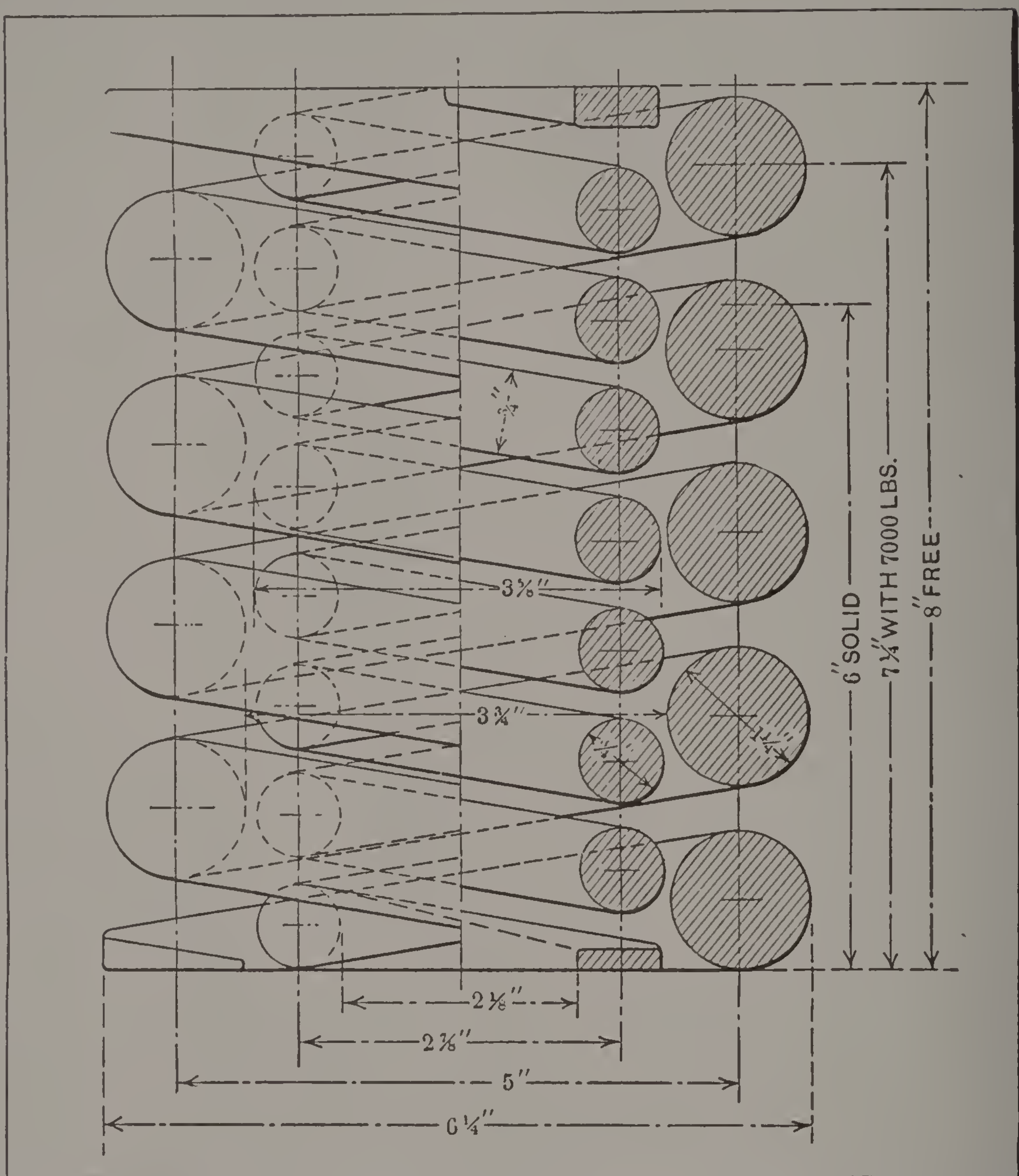


Fig. 19. A Concentric Group showing what is meant by "Solid," "Loaded" and "Free Heights." The Clearance between Coils is usually one-sixteenth inch

Equation of d_1 for Single Coil Spring

The value of d_1, or the diameter of the inner (in this case the only) bar may be obtained by writing equation (16) simply as:

$$\frac{\pi\ d_1^2}{4} = \frac{2\ P}{S}\ r \tag{24}$$

which may be readily solved for d_1 after making the proper numerical substitutions of the other quantities, thus:

$$d_1^2 = \frac{8\ P}{\pi\ S}\ r \tag{25}$$

Equation of d_1 for Double Coil Spring

If there are reasons for desiring two coils in the spring, equation (21) gives

$$d_2 = \left(\frac{r+1}{r-1}\right) d_1 + \frac{c}{r-1} \tag{26}$$

and from equation (16)

$$\frac{\pi}{4}(d_1^2 + d_2^2) = \frac{2P}{S} r \tag{27}$$

Substituting the above value of d_2 in this gives an equation which after substitution of constants may be readily solved for d_1, after which d_2 and the outside diameters may be readily found. This substitution results in

$$d_1^2 + \left[\left(\frac{r+1}{r-1}\right) d_1 + \frac{c}{r-1}\right]^2 = \frac{8P}{\pi S}(r) \tag{28}$$

Equation of d_1 for Triple Coil Spring

If now three coils are desired, from equation (21) as before:

$$d_2 = \frac{r+1}{r-1} d_1 + \frac{c}{r-1} \tag{29}$$

and from equation (22)

$$d_3 = \frac{r+1}{r-1} d_2 + \frac{c}{r-1} \tag{30}$$

whence

$$d_3 = \left(\frac{r+1}{r-1}\right)^2 d_1 + \frac{r+1}{(r-1)^2} c + \frac{c}{r-1} \tag{31}$$

Then from equation (16)

$$\frac{\pi}{4}(d_1^2 + d_2^2 + d_3^2) = \frac{2P}{SG} r \tag{32}$$

whence

$$d_1^2 + \left(\frac{r+1}{r-1} d_1 + \frac{c}{r-1}\right)^2 + \left(\frac{(r+1)^2}{(r-1)^2} d_1 + \frac{r+1}{(r-1)^2} c + \frac{c}{r-1}\right)^2 = \frac{8P}{\pi S} r \tag{33}$$

Equation of d_1 for any Number of Coils

From equation (14) it is apparent that if there be n number of coils, the nth value of d, or d_n, is always

$$d_n = \frac{r+1}{r-1} d_{n-1} + \frac{c}{r-1} \tag{34}$$

The general formula for d_1 may therefore always be written, although every additional coil adds greatly to the complexity of the final expression.

Obstacles in the Design of Concentric Coils

The increasing complexity of the equation of d_1, offers an obstacle to the solution of the multi-coiled springs on strictly mathematical lines. A still greater obstacle in the use of the formulas deduced lies in the fact that commercially it is found economically practical to use only such sizes of bars as are commonly rolled by the mills. In the absence of tables giving the properties of various spring coils from which a selection may be readily made, it is believed that the above formulas, (25), (28), (33) and other similar formulas which may be readily deduced, will serve as guides to the best commercial sizes to use, which sizes being once determined may then be investigated and their future combined action ascertained with certainty. To make the manner of proceeding clearer, assume a definite problem.

Solution of Problem by Foregoing Formulas

Problem: To ascertain the proper coils to use to support 35,464 pounds at 5.022 inches solid height, the free height to be 6.625 inches.

From equation (7) the spring index is

$$r = \sqrt{\frac{(6.625 - 5.022)\ 12{,}600{,}000}{3.1416 \times 80{,}000 \times 5.022}}$$

whence $r = 4$, closely.

Size of Bar for One Coil Spring:

By equation (25)

$$d_1^2 = \frac{8 \times 35464 \times 4}{3.1416 \times 80{,}000}$$

whence $d_1 = 2\frac{1}{8}$ inches.

Therefore $D = 4d_1 = 8\frac{1}{2}$ inches
and $D'' = 5d_1 = 10\frac{5}{8}$ inches.

Size of Bar for Two Coil Springs:

Assume the usual clearance of 1/16 inch between coils, whence $c = \frac{1}{8}$ inch. From equation (28)

$$d_1^2 + \left(\frac{5}{3} d_1 + \frac{1}{24}\right)^2 = \frac{.8 \times 35464 \times 4}{3.1416 \times 80{,}000}$$

or

$$\frac{34}{9} d_1^2 + \frac{10}{72} d_1 + \frac{1}{576} = 4.5156$$

whence $d_1 = 1.07$ inch and $d_2 = 1.825$ inch. Therefore $D_1'' = 5.35$ inches, and $D_2'' = 9.125$ inches.

Result of Adopting Bars of Commercial Sizes

The closest commercial sizes to the above decimal solutions would then be

$$d_1 = 1\tfrac{1}{16} \text{ inch, and } d_2 = 1\tfrac{13}{16} \text{ inch, and}$$

$$D_1'' = 5\tfrac{3}{8} \text{ inches and } D_2'' = 9\tfrac{1}{8} \text{ inches}$$

The actual value of c is then (see notation on page 41).

$$D_2' - D_1'' = 5\tfrac{1}{2} - 5\tfrac{3}{8} = \tfrac{1}{8} \text{ inch}$$

Now turn the investigation to the two coils which have been selected. The spring index of the inner coil will be

$$r_1 = \frac{D_1}{d_1} = \frac{4\frac{5}{16}}{1\frac{1}{16}} = \frac{69}{17} = 4\text{-}1/17$$

Of the outer coil,

$$r_2 = \frac{D_2}{d_2} = \frac{7\frac{5}{16}}{1\frac{13}{16}} = \frac{117}{29} = 4\text{-}1/29$$

It may now be seen that although the design was based on a constant spring index, the limitations of practice and economy have rendered it impossible to maintain this ideal condition. As the spring indexes of the inner and outer coils are of different values, it is known at once that the deflections and lengths of bar will not be identical for the same solid height. This means that commencing with the same free height and compressing to the same height will cause one coil (that having the least value of spring index) to be stressed higher than the other.

If the value of the spring index has been diminished only slightly from that assumed, it is a safe assumption that the fiber stress will be increased but slightly beyond that assumed, in which case it is not necessary to calculate the actual stresses, but the real capacities may be arrived at directly by basing the calculations upon the modulus of elasticity. It is more satisfactory, however, to ascertain the fiber stresses also, and where the value of the spring index has been considerably altered such a course is imperative in order to keep within safe limits of stress.

Solution of Actual Problem—Stresses

The results will be the same whether similar free heights be taken and compressed to the same maximum fiber stress, or whether a beginning be made with the same solid height extending to the same maximum stress; the maintenance of a uniform final stress results in final heights which are not uniform. Instead of different final heights the usual practice is to use uniform free and solid heights, with the result that each coil is then stressed differently as pointed out before.

In this case the actual stress in each coil is found by the formula

$$S = \frac{G f}{\pi h}\left(\frac{d}{D}\right)^2$$

which is simply an expression of the fact that where the material used, and the free and solid heights are uniform, the stress varies inversely as the square of the spring index.

In the particular problem at hand, the stress in the inner coil would then be

$$S_1 = 4{,}010{,}695 \frac{6.625 - 5.022}{5.022} \left(\frac{17}{69}\right)^2 = 77{,}700 \text{ pounds}$$

and in the outer

$$S_2 = 4{,}010{,}695 \frac{6.625 - 5.022}{5.022} \left(\frac{29}{117}\right)^2 = 78{,}600 \text{ pounds}$$

Or, since

$$S_1 : S_2 :: \left(\frac{1}{r_1}\right)^2 : \left(\frac{1}{r_2}\right)^2$$

then

$$S_2 = \left(\frac{r_1}{r_2}\right)^2 S_1$$

or

$$S_2 = \left(\frac{29}{117}\right)^2 \left(\frac{69}{17}\right)^2 77{,}700 = 78{,}600 \text{ pounds}$$

which is the same as before.

The stresses being known, the load on each coil may now be solved by the following formula:

$$P = \frac{\pi S d^3}{8 D}$$

Solution of Actual Problem—Capacities

When the deviation from the assumed index is slight, the variation in the maximum stress will be correspondingly small, and the experienced designer is therefore safe in proceeding to estimate the capacity of his spring directly from dimensions and without reference to actual stresses. In this case use the formula

$$P = \frac{G f d^5}{8 h D^3}$$

or, where G is 12,600,000 for steel springs

$$P = 1{,}575{,}000 \frac{f d^5}{h D^3}$$

This would give for the inner coil

$$P_1 = 1{,}575{,}000 \frac{6.625 - 5.022}{5.022} \frac{(1.0625)^5}{(4.3125)^3} = 8490$$

and for the outer,

$$P_2 = 1{,}575{,}000 \frac{6.625 - 5.022}{5.022} \frac{(1.8125)^5}{(7.3125)^3} = 25150$$

The capacity of the two coils together will then be

$$P_1 + P_2 = P = 33{,}640 \text{ pounds}$$

Some idea of the difference which exists in the theoretical and practical design may now be gained from Table I, which makes a detailed comparison.

Table I. Comparison Between Estimated and Actual Coil Spring Results

	Estimated	Actual
Free height (Assumed same as actual)	6.625	6.625
Solid height (Assumed same as actual)	5.022	5.022
Deflection (Assumed same as actual)	1.603	1.603
Stress, inner coil	80,000	77,700
Stress, outer coil	80,000	78,600
Capacity	35,464	33,640
Diameter inner bar	1.07	1.0625
Diameter outer bar	1.825	1.8125
Outside diameter inner coil	5.35	5.375
Outside diameter outer coil	9.125	9.125

Limitations of Concentric Grouping

It is now apparent that in a spring concentrically arranged the inner bars are properly the smaller, and the greatest load is naturally upon the outer. There is a point, however, beyond which more inner coils will cease to be of advantage owing to the small gain in capacity. The addition of outer coils is also soon limited by the impossibility of coiling and tempering large bars. It is therefore evident that the load which may be carried by the concentric group is limited.

Spring Plate Groups

Where greater capacity is desirable than can be obtained by concentric grouping, several single coils, or several concentric groups, may be held together between spring plates of malleable cast iron or pressed steel. Such groups naturally offer greater stability than concentric groups; but, where the concentric group affords sufficient capacity and stability it should be used, as it is more economical of space and does not necessitate the use of spring plates to hold the different coils together. As the load should be supported firmly upon the center of the unit, the group should be arranged with such symmetry that the supporting forces, or spring resistances, will balance about any axis.

The designing of groups of this kind consists in the simple operation of dividing the load into as many parts as there will be units in the group. Then, maintaining the desired free heights and solid heights, and hence the same constant spring index, proceed to design the separate units in the manner just presented for the simple concentric. Ordinarily much time and labor may be saved by remembering that halving the diameters of bar and coil reduces the capacity and weight to one-quarter, but does not affect length of bar or deflection of coil. This is due to the fact that this really halves the spring index with effect as indicated in formulas (1), (2), (3) and (4) page 38.

TABLE II. COMPARISON OF FOUR COIL SPRING GROUPS FOR SAME CAPACITY

	Description		O. D. or Equivalent, Ins.	Free, Ins.	Solid, Ins.	Bars, Ins.	Spring Index	Capacity, Pounds	Weight per Inch Solid Height, Pounds	Length of Bar per Inch Height, Ins.
Group A	Four similar coils in spring plates		11 to 12	$6\frac{5}{8}$ between plates	5.09 betw'n plates	$1\frac{1}{16}$	4	4 × 8866 = 35,464	4 × 3.1569 = 12.63	12.57
Group B	Single coil		$10\frac{5}{8}$	$6\frac{5}{8}$	5.09	$2\frac{1}{8}$	4	35,464	12.63	12.57
Group C	Double coil, concentric	Outer	$9\frac{3}{8}$	$6\frac{5}{8}$	5.09	$1\frac{7}{8}$	4	37,611	9.83	12.57
		Inner	5	$6\frac{5}{8}$	5.09	1	4	7,853	2.80	12.57
								35,464	12.63	
Group D	Triple coil, concentric	Outer	$8\frac{3}{4}$	$6\frac{5}{8}$	5.09	$1\frac{3}{4}$	4	24,076	8.56	12.57
		Medium	5	$6\frac{5}{8}$	5.09	1	4	7,854	2.80	12.57
		Inner	$2\frac{13}{16}$	$6\frac{5}{8}$	5.09	$\frac{9}{16}$	4	2,485	0.88	12.57
								34,415	12.24	

As illustrating clearly the comparison between different but equivalent springs, Table II is included in this article, showing four equivalent arrangements. The reader will note that as more coils are used a more compact design becomes possible for the concentric arrangements. The size of the bars reduces also, which makes possible better tempering.

CHAPTER VI.

CONICAL HELICAL SPRINGS

On account of their physical characteristics, conical helical springs divide themselves into two distinct classes, according to their use. The formulas applicable to a spring of this type used as a compression spring are not at all applicable if the spring is to be used as an extension spring. This is more obvious if we note that the safe load which an extension spring will carry is governed by the capacity of the largest or weakest coil, which condition is reversed in the compression state, where the spring retains its flexibility until the load becomes great enough to close up the smallest or strongest coil. It is the object of this article to develop the formulas applicable to the various types of conical helical springs. Round bar coils only will be considered.

Notation—Dimensions in Inches, Weights in Pounds

S = stress,
G = modulus of torsional elasticity,
f = deflection,
f_x = deflection under load P_x
H = free height,
h = solid height, assumed to equal $d \times N$,
y = solid height at any convenient coil,
P = capacity of spring,
P_x = any load not exceeding P,
P_1 = capacity of largest coil,
P_2 = capacity of smallest coil,
D_1 = mean diameter of largest coil,
D_2 = mean diameter of smallest coil,
d = diameter of bar from which coil is made,
A = mean radius of largest coil,
B = apex height of cone,
C = mean radius of smallest coil,
N = number of coils,
p = average horizontal pitch of coils,
w = weight of one cubic inch of steel,
l = length of bar in spring,
W = weight of spring,
x = mean radius of any convenient coil.

Deflection and Capacity of Extension Spring

It is obvious that P_x in this case must be understood to be not greater than the capacity of the weakest coil, inasmuch as a greater value of P_x would distort the spring beyond the realm of rational formulas. A conical spring is composed of an infinite number of elementary cylindrical coils, each element being in itself a uniform diameter helical spring, and each element differing from its neighbor in the one respect that each successive element has a mean diameter

infinitesimally less than its first neighbor, and likewise infinitesimally greater than its neighbor on the other side. These increments of change in the mean diameter result in corresponding increments of change in the deflection of the successive elements, and being all governed by the one general expression for deflection of cylindrical helical springs, may be added together by resorting to calculus. Since

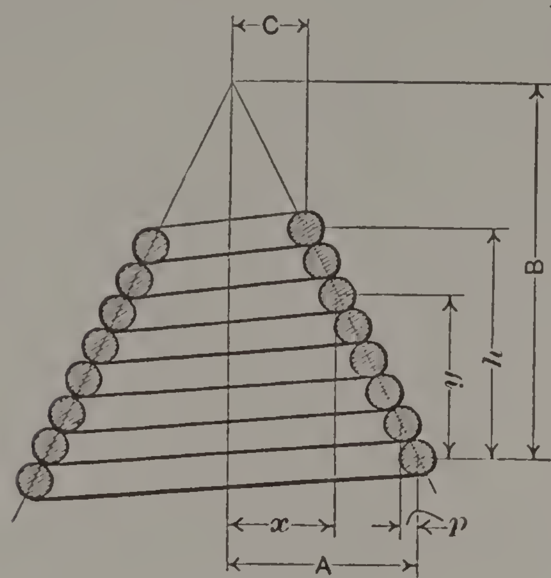


Fig. 20

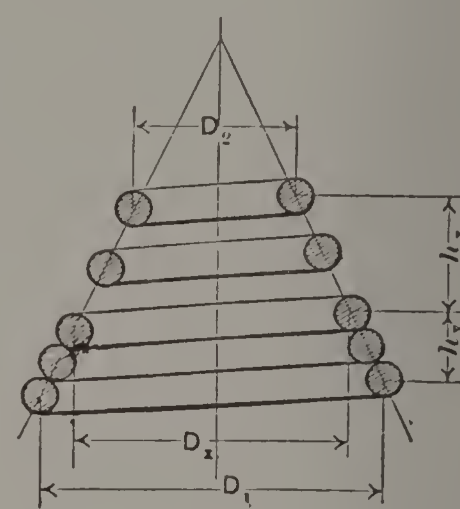


Fig. 21

Diagrams for the Derivation of Spring Formulas

the increments of change in the mean diameter are in this case in proportion to the increments of change in the solid height, it follows that the increments of change in the deflection also follow those of the solid height, and that we may expect to arrive at the summation of the deflection through a summation of the increments of change in the varying solid height, which for successive elementary cylindrical coils increases from 0 to its maximum h.

Now the expression for deflection in cylindrical coil springs is:

$$f = \frac{\pi S}{G} \left(\frac{D}{d} \right)^2 h$$

and for capacity:

$$P = \frac{\pi S d^3}{8 D}$$

The value of D is here, however, the variable, represented by $2x$. Therefore, in each elementary cylinder:

$$\frac{f_x}{f} = \frac{P_x}{P}, \text{ or } f_x = \frac{8 P_x (2x)^3 h}{G d^5} = \frac{64 P_x x^3 h}{G d^5}$$

Further,

$$\delta f_x = \frac{64 P_x}{G d^5} x^3 \delta y$$

$$f_x = \int_0^h \frac{64 P_x}{G d^5} x^3 \delta y = \frac{64 P_x}{G d^5} \int_0^h x^3 \delta y.$$

But, as shown in Fig. 20, $\frac{B}{y}=\frac{A}{A-x}$; whence $x=A-\frac{A\,y}{B}$

$$\text{Hence } x^3=A^3\left(1-\frac{3\,y}{B}+\frac{3\,y^2}{B^2}-\frac{y^3}{B^3}\right)$$

Hence,

$$f_x=\frac{64\,P_x\,A^3}{G\,d^5}\int_0^h\left(1-\frac{3\,y}{B}+\frac{3\,y^2}{B^2}-\frac{y^3}{B^3}\right)\delta\,y$$

$$f_x=\frac{64\,P_x\,A^3}{G\,d^5}\left(h-\frac{3\,h^2}{2\,B}+\frac{h^3}{B^2}-\frac{h^4}{4\,B^3}\right)$$

Also, from Fig. 20, we have: $B=\frac{A\,h}{A-C}$, whence:

$$f_x=\frac{64\,P_x\,A^3}{G\,d^5}\left(h-\frac{3\,h^2\,(A-C)}{2\,A\,h}+\frac{h^3\,(A-C)^2}{A^2\,h^2}-\frac{h^4\,(A-C)^3}{4\,A^3\,h^3}\right)$$

But $A=\frac{D_1}{2}$ and $C=\frac{D_2}{2}$. Hence,

$$f_x=\frac{8\,P_x\,D_1^3\,h}{G\,d^5}\left(1-\frac{3\,(D_1-D_2)}{2\,D_1}+\frac{(D_1-D_2)^2}{D_1^2}-\frac{(D_1-D_2)^3}{4\,D_1^3}\right)$$

$$=\frac{8\,P_x\,D_1^3\,h}{G\,d^5}\left(\frac{D_1^3+D_1^2\,D_2+D_1\,D_2^2+D_2^3}{4\,D_1^3}\right)$$

$$=\frac{2\,P_x\,h}{G\,d^5}\,(D_1^3+D_1^2\,D_2+D_1\,D_2^2+D_2^3)=\frac{2\,P_x\,h\,(D_1^4-D_2^4)}{G\,d^5\,(D_1-D_2)}$$

$$=\frac{P_x\,(D_1^4-D_2^4)}{G\,d^4\dfrac{(D_1-D_2)\,d}{2\,h}}$$

Observe now that the expression in the denominator is the average horizontal pitch p (see Fig. 20) of the coils:

$$p=\frac{\dfrac{D_1-D_2}{2}}{N};\ \text{but } N=\frac{h}{d};\ \text{hence, } p=\frac{(D_1-D_2)\,d}{2\,h}$$

Therefore,

$$f_x=\frac{P_x\,(D_1^4-D_2^4)}{G\,p\,d^4}$$

This, then, is the formula for the deflection of any extension type conical helical spring under load P_x, the value P_x not exceeding the capacity of the weakest coil, equivalent to the capacity of a cylindrical helical spring of a mean diameter equal to D_1 and bar diameter equal to d.

If the value of P_x equals the capacity of P of this spring, we have:

$$P = P_x = \frac{\pi\, S\, d^3}{8\, D_1}$$

Substituting this value of P_x, we have the total deflection:

$$f = \frac{\pi S\, (D_1^4 - D_2^4)}{8\, G\, p\, d\, D_1}$$

which is the final formula for the total deflection of the extension conical helical spring.

It will be noticed that the previous discussion is based on the assumption that the solid height of the spring is equal to the diameter of the bar times the number of coils in the conical spring. However, as one coil seats within the other, the solid height is really less than that assumed; the solid height becomes less as the taper of the coil becomes greater, until for a true spiral spring the solid height is reduced to the diameter of the bar. The actual deflection is dependent on the "slant height" of the conical coil which is equal to $d \times N$, as assumed for the solid height, rather than upon the actual vertical height. Due to this assumption, the changing of the angle of the cone formed by the spring does not change the actual deflection so long as the value of the slant height itself is not changed. Inasmuch as the above discussion is based on the slant height, the actual solid height of the spring and also the free height of the spring should be corrected by deducting from the value of these heights the difference between the slant height and the solid height. Stated briefly, the assumption that the solid height is equal to $d \times N$ is necessary in order to obtain the correct value of the deflection; but after this deflection has been obtained, correction should be made for this assumption.

Deflection and Capacity of Compression Spring

The deflection of a compression spring of this type is a fundamental problem of the same type. In this case, however, the summation is not the summation of increments of deflection under a uniform load and varying stresses, but the summation of increments of deflection when each elementary spring is stressed to a maximum, and hence under uniform stress.

In this case $f = \dfrac{\pi S}{G}\left(\dfrac{D}{d}\right)^2 h$, becomes $f = \dfrac{\pi S}{G}\left(\dfrac{2x}{d}\right)h$

$$\delta f = \frac{\pi S}{G}\left(\frac{2x}{d}\right)^2 \delta y \text{ and } f = \int_0^h \frac{\pi S}{G}\left(\frac{2x}{d}\right)^2 \delta y$$

But $x = A - \dfrac{A\, y}{B}$. Hence:

$$f = \frac{4\,\pi\, S\, A^2}{G\, d^2}\int_0^h \left(1 - \frac{2\,y}{B} + \frac{y^2}{B^2}\right)\delta y$$

$$= \frac{4\,\pi\, S\, A^2}{G\, d^2}\left(h - \frac{h^2}{B} + \frac{h^3}{3\, B^2}\right)$$

But $B = \dfrac{A\,h}{A - C}$, and $A = \dfrac{D_1}{2}$, and $C = \dfrac{D_2}{2}$. Hence the expression above can be transformed to:

$$f = \frac{\pi\,S\,h}{3\,G\,d^2}\,(D_1^2 + D_1\,D_2 + D_2^2)$$

$$f = \frac{\pi\,S\,h\,(D_1^3 - D_2^3)}{3\,G\,d^2\,(D_1 - D_2)}, \text{ and since } p = \frac{(D_1 - D_2)\,d}{2\,h}.$$

$$f = \frac{\pi\,S\,(D_1^3 - D_2^3)}{6\,G\,p\,d}$$

which expresses the total compression for a conical helical compression spring.

The capacity, solid, equals that of the smallest coil, or:

$$P = \frac{\pi\,S\,d^3}{8\,D_2}$$

Deflection of Compression Spring for Given Loads

If the given load is less than P_1, the entire spring remains flexible, and the formula for deflection is the same as that derived for an extension spring, the condition of varying stress still being present.

If, however, the load exceeds P_1, then a portion of the spring will become solid. The division point may be found, for $P_x = \dfrac{\pi\,S\,d^3}{8\,D_x}$, and since the load in cylindrical coils varies inversely as D:

$$\frac{D_x}{D_2} = \frac{P_2}{P_x}, \text{ whence } D_x = \frac{D_2 P_2}{P_x}$$

The deflection of the two portions of the spring should now be considered separately and added, using the two final formulas just developed.

Fig. 21 shows a graphical illustration of the divided spring. Let p_x be the average horizontal pitch of the unclosed portion, above D_x. Let p_y be the average horizontal pitch of the solid portion, below D_x.

The total deflection is then:

$$f = \frac{\pi\,S\,(D_1^3 - D_x^3)}{6\,G\,d\,p_y} + \frac{\pi\,S\,(D_x^4 - D_2^4)}{8\,G\,d\,D_x p_x}$$

In Fig. 21, h_y is the height, solid, of the solid portion of the spring. Its value is derived thus:

$$\frac{h_y}{h} = \frac{D_1 - D_x}{D_1 - D_2}, \text{ whence } h_y = h\,\frac{D_1 - D_x}{D_1 - D_2}.$$

Bar Length for Conical Spring

Fundamentally, $l = \pi \left(\frac{D}{d}\right) h$, so that we have in a conical spring

$$l = \delta \pi \left(\frac{2x}{d}\right) \delta y, \text{ or}$$

$$l = \frac{2\pi}{d} \int_0^h x \, \delta y = \frac{2A\pi}{d} \int_0^h \left(1 - \frac{y}{B}\right) \delta y = \frac{2A\pi}{d} \left(h - \frac{h^2}{2B}\right)$$

And since $A = \frac{D_1}{2}$ and $B = \frac{Ah}{A - C}$, and $C = \frac{D_2}{2}$, we get:

$$l = \frac{\pi h}{d} \left(\frac{D_1 + D_2}{2}\right)$$

Weight of Conical Spring

Fundamentally, $W = \frac{l \pi d^2 w}{4}$. Hence

$$W = \frac{\pi^2 d h w}{8} (D_1 + D_2)$$

Summary of Formulas

Summarizing and substituting for S a value of 80,000 pounds per square inch, and for G a value of 12,600,000 we have:

Deflection; $f = 0.002493 \frac{D_1^4 - D_2^4}{p \, d \, D_1}$ for extension spring

$f = 0.003324 \frac{D_1^3 - D_2^3}{p \, d}$ for compression spring

Capacity; $P = 31{,}416 \frac{d^3}{D_1}$ for extension spring

$P = 31{,}416 \frac{d^3}{D_2}$ for compression spring

Weight; $W = 0.35 \, dh \, (D_1 + D_2)$ in each case

Bar length; $l = 1.571 \frac{h}{d} (D_1 + D_2)$ in each case

Numerical Examples

Example: Compression spring, $D_1 = 4\ 9/16$ inches; $D_2 = 3\ 9/16$ inches; $d = 1\ 1/16$ inch; $h = 7\ 1/16$ inches.

$$\text{Then } N = \frac{h}{d} = 6.65;\quad \frac{D_1 - D_2}{2} = 0.5;\quad p = \frac{D_1 - D_2}{2N} = 0.075.$$

$$f = 0.003324\, \frac{(4\tfrac{9}{16})^3 - (3\tfrac{9}{16})^3}{0.075 \times 1\tfrac{1}{16}} = 2\tfrac{1}{16} \text{ approximately.}$$

See the accompanying table of cubes, and also other powers of numbers required in conical spring calculations.

$$H = 7\ 1/16 + 2\ 1/16 = 9\ 1/8 \text{ inches.}$$

Example: Same spring in extension.

$$f = 0.002493\, \frac{(4\tfrac{9}{16})^4 - (3\tfrac{9}{16})^4}{0.075 \times 1\tfrac{1}{16} \times 4\tfrac{9}{16}} = 1\tfrac{7}{8} \text{ approximately.}$$

$$H \text{ (extended height)} = 7\tfrac{1}{16} + 1\tfrac{7}{8} = 8\tfrac{15}{16} \text{ inches.}$$

As might have been expected, the free height for the compression type is greater than the possible extended length for the extension type. This is because sufficient load to fully stress the smaller or stronger coils cannot be applied without distorting the extension spring, whereas the coils may all be stressed to maximum stress in the compression type, the closing of the coils solidly together protecting the spring from over-stress.

Reversion to Cylindrical Helical Springs

It will be noted that in each of our final formulas we have introduced the factor $(D_1 - D_2)$ and the value of $p = \left(\frac{D_1 - D_2}{2h}\right) d$. This has been done to leave the formulas in as simple a form as possible. Note, however, that if D_1 and D_2 each be taken as equal to D_1 or to each other, the formulas in each case revert to the fundamental cylindrical helical spring formulas, as should be expected.

Substituting $D_1 = D_2 = D$ in the extension formula

$$f_x = \frac{2 P_x h}{G d^5} (D_1^3 + D_1^2 D_2 + D_1 D_2^2 + D_2^3), \text{ we have}$$

$$f_x = \frac{8 P_x D^3 h}{G d^5}$$

This is the fundamental formula for the deflection of an extension helical cylindrical spring under any load P_x, derived directly from the formula for load:

$$P = \frac{\pi S d^3}{8 D}$$

in which S has been replaced by its value in the formula

$$f = \frac{\pi S}{G} \left(\frac{D}{d} \right)^2 h \text{ giving } P = \frac{G f d^5}{8 h D^3}$$

from which $f = \frac{8 P D^3}{G d^5} h$

Substitute, again, $D_1 = D_2 = D$ in the compression formula:

$$f = \frac{\pi S H}{3 G d^2} (D_1^2 + D_1 D_2 + D_2^2), \text{ and we have}$$

$$f = \frac{\pi S}{G} \left(\frac{D}{d} \right)^2 h$$

the fundamental formula for the deflection of a compression helical cylindrical spring.

The reason for this comparison between the formulas for cylindrical helical springs and conical springs, is particularly, to bring out the fact that, while each of the conical formulas are thus shown to revert to the same form when $D_1 = D_2$, yet the conical spring formulas themselves, for extension and compression springs, are different. As already mentioned, one is an expression for deflection under a given load, regardless of stress, while the other is the expression for deflection under a uniform maximum stress. The former condition is that of the conical extension spring, while the latter is that of the compression type.

Auxiliary Formulas

The expressions for deflection and capacity are the main formulas for all helical springs; from these are developed such other formulas as may be desired. In this particular case, care must be taken in making such further developments, if using the expressions $D_1^4 - D_2^4$ and $D_1^3 - D_2^3$, to note that resulting formulas will not revert to simple cylindrical helical formulas, because of the fact that a zero quantity has been introduced when D_1 becomes equal to D_2.

Therefore, further formulas are based on the longer but primary formulas which we have arrived at before the introduction of the quantity $D_1 - D_2$.

The Ratio Between Free and Solid Heights

Since $H = f + h$, we have, for compression springs:

$$H = \frac{\pi S h}{3 G d^2} (D_1^2 + D_1 D_2 + D_2^2) + h$$

$$= h \left[1 + \frac{\pi S}{G} \left(\frac{D_1^2 + D_1 D_2 + D_2^2}{3 d^2} \right) \right]$$

$$= h \left[1 + \frac{\pi S}{G} \left(\frac{D_1^3 - D_2^3}{3 d^2 (D_1 - D_2)} \right) \right]$$

In a similar way, for extension springs:

$$H = h \left[1 + \frac{2 P_x}{G} \left(\frac{D_1^3 + D_1^2 D_2 + D_1 D_2^2 + D_2^3}{d^5} \right) \right]$$

$$= h \left[1 + \frac{2 P_x}{G} \left(\frac{D_1^4 - D_2^4}{d^5 (D_1 - D_2)} \right) \right]$$

By introducing the factor $(D_1 - D_2)$ the formulas are thus simplified as before, so that they may be readily solved with a table of cubes and fourth powers.

Deflection When Only Free Height is Given

Considering first the compression type, we substitute the value of h as found from the formulas in the last paragraph in the general formula for f. Hence:

$$f = \frac{H}{1 + \frac{G}{\pi S} \left(\frac{3 d^3}{D_1^2 + D_1 D_2 + D_2^2} \right)} = \frac{H}{1 + \frac{G}{\pi S} \left(\frac{3 d^2 (D_1 - D_2)}{D_1^3 - D_2^3} \right)}$$

In a similar way, for extension coils:

$$f = \frac{H}{1 + \frac{G}{2 P_x} \left(\frac{d^5}{D_1^3 + D_1^2 D_2 + D_1 D_2^2 + D_2^3} \right)}$$

$$= \frac{H}{1 + \frac{G}{2 P_x} \left(\frac{d^5 (D_1 - D_2)}{D_1^4 - D_2^4} \right)}$$

General Considerations

In the compression type it is sometimes desirable to fix the heights, both free and solid, and afterwards ascertain the resulting capacity. If the heights so fixed exceed the allowable deflection by the compression formula, the spring will not return to its original free height. In other words, it will have taken a set. If the difference in heights is less than that of the compression formula, it cannot be assumed that there will be a uniform stress throughout the spring when solid, as there would have been had the spring been built to the highest free height possible, and the capacity will not then be in proportion to the deflection. If the deflection, for instance, is one-half of the formula deflection, the capacity will not necessarily be one-half that of the strongest coil, instead of equal to that of the strongest coil. This type then appears indeterminable for capacity, the difficulty being to so pitch the coils as to assure uniform stress when the spring is solid. This difficulty does not present itself in cylindrical coils as we have a uniform stress at solid height.

Uniform stress at solid height in a conical spring requires a pitch of coils in proportion to the deflection of same at maximum stress, or, which is the same thing, in proportion to the diameters of the various elements. As the diametrical increase per unit of bar length is not a straight line formula, the pitch of coils necessary to gain uniform stress when solid would have to follow the law of a definite curve. While it may be possible to develop a machine which will so pitch these elementary coils, yet the demand does not seem to have developed such a machine.

Where the deflection is made originally greater than the maximum stress will allow, the first compressions of the hardened spring will reduce the deflection to the maximum which the steel will stand. Thus, in such a case there is an assurance of uniform stress.

The laws governing the action of grouped cylindrical helical springs apply likewise to grouped conical springs. Briefly, the design should maintain the same free and solid heights throughout, which means that for all coils in the group the $\frac{D_1}{d}$ ratio should be the same, and the $\frac{D_2}{d}$ ratio should likewise be the same for all coils.

CHAPTER VII.

WIRE SPRINGS

Wire is classified by gage, a certain gage meaning a wire that will pass through a standard sized hole or slot. These holes or slots are numbered; thus a No. 10 wire will pass through the No. 10 hole in the gage. From this it will be clear that wires are not identified by their actual diameters as in the case of round bar stock. If there were only one wire gage in general use, this method of specifying wire sizes would be less confusing; but unfortunately there are several gage systems which were originally developed by workers in different trades or shops, and each of these is in more or less general use. The more common of these wire gages are: (1) American, or Brown & Shape; (2) Birmingham, or Stubb's iron wire; (3) Washburn & Moen; (4) Stubb's steel wire; (5) American screw gage; and (6) steel music wire. As a complete series of wire sizes are made in accordance with each of these systems, and as only the more widely used systems are mentioned in the preceding list, it will be evident that the different sizes of wire are numerous and that these sizes overlap each other.

Impracticability of a Complete Table

In arranging a table of springs which are made from rods, the table may be laid out according to the sizes of the rods, *i. e.*, 1/4, 5/16, 3/8 inch in diameter, etc. Then the properties of the springs may be tabulated under each size of rod, according to the outside diameter of the spring, *i. e.*, 4 inches, 4 1/16 inches, 4 1/8 inches, etc. In such cases intervals of 1/16 inch in the outside diameter of the spring or in the diameter of the bar from which the spring is made, is the smallest variation required for all ordinary cases. No such method can be followed in working out a table of wire springs, however, because the different wire gages in common use are too numerous and the variations in the diameters of the springs are infinite, the spaces being very erratic and often consisting of infinitesimally small differences. Moreover, the variation of any dimension by even 1/32 inch may be entirely too great when compared with the size of the spring, as some wire springs are so small that it requires several thousand of them to weigh one ounce. The best that can be done is to resort to a makeshift and develop an approximate table which will give a good idea of the properties of any given spring. In addition to the trouble resulting from different wire gages in common use, a further difficulty is encountered owing to the fact that springs are made of all kinds of material—such as aluminum, brass, bronze, steel, etc.—and there are no standard grades of wire for this purpose, as in the case of rods where the Pennsylvania Railroad specifications for

spring steel are recognized as a standard. On this account we can only base the data in a wire spring table on a certain specified fiber stress and leave it to the engineer to use that as a basis where a different fiber stress must be used.

The Spring Index

No matter what the size of the wire or the diameter of the spring may be, it is always possible to arrive at the value of the ratio which we have termed the "spring index." This ratio is given by the following:

$$\text{Spring index} = \frac{D}{d}$$

where D = mean diameter of spring;
d = diameter of wire.

A wire spring table containing all values of the spring index $\frac{D}{d}$ would, of course, be infinitely long and include infinitely small differences, so that any practical table must omit many possible values. Consequently, reference to the table will often result in failure to find the exact value of the spring index $\frac{D}{d}$ that is wanted. In such cases the nearest value will serve to give a fair idea of the actual spring properties, and will generally be close enough to answer all practical purposes. In order to use the spring table which is presented in this connection, the first step is to calculate the value of $\frac{D}{d}$ for the required spring, and also the value of d^2. The second and fourth columns of the table contain constants which are multiplied directly by the solid height of the spring, but the values in the third and fifth columns of the table must be multiplied by d^2. If the weight of the spring is required, it is also necessary to multiply by the solid height of the spring. The results presented in the table are based upon a value of 80,000 pounds per square inch for the fiber stress and a torsional modulus of 12,600,000.

To make the method of procedure quite clear, we will calculate the properties of a wire spring to be made of spring steel which has a safe fiber stress of 80,000 pounds per square inch, the diameter d of the wire being 1/16 inch and the mean diameter D of the spring, 3/16 inch. The solid height of the spring is to be 1¼ inch. It is required to know: (1) the length of wire necessary to make the spring; (2) the weight of the wire in the spring; (3) the free height of the spring; and (4) capacity of spring. With this, we find,

$$\text{Spring index } \frac{D}{d} = \frac{3/16}{1/16} = 3$$

$$d^2 = [1/16]^2 = \frac{1}{256}.$$

Now referring to the table (Page 63) and a value of 3 for the spring index, $\frac{D}{d}$, we find the required length of wire to make the spring to be (from the second column of the table):

$$1\tfrac{1}{4} \times 9.4248 = 11.78 \text{ inches.}$$

From the third column, the weight of this wire is found to be:

$$\frac{1}{256} \times 2.0973 \times 1\tfrac{1}{4} = 0.0102 \text{ pound.}$$

From the fourth column, the free height of the spring is found to be:

$$1\tfrac{1}{4} \times 1.1795 = 1.47 \text{ inch.}$$

From the fifth column, the capacity of the spring is found to be:

$$\text{Capacity } \frac{1}{256} \times 10{,}472 = 41 \text{ pounds per coil.}$$

Initial Tension

Springs are often made with an "initial tension" which causes the coils to be drawn tightly together. This result is secured by twisting the wire, a common example of a spring of this type being the ordinary screen door spring. Such springs will not begin to deflect as soon as the load is applied, it being necessary to first overcome the initial tension already in the spring. With springs of this type it is possible to load to the maximum capacity without obtaining a corresponding deflection of the spring.

Methods Used in Spring Manufacture

Wire springs are made of a great variety of materials, and the wire is generally sold under a trade name. The manufacturer is very willing to guarantee his wire of certain grades, but equally unwilling to guarantee other grades of wire which he makes. In all cases, however, he either cannot or will not state any of the physical properties of his product, contenting himself by saying it is "extremely uniform." Some manufacturers actually disclaim the possession of such knowledge, while others are frank in stating that exact analyses and corresponding characteristics are regarded as valuable trade secrets. Some manufacturers of both wire and wire springs, even go so far as to state that they make certain grades of wire for their own use and that they will not sell such wire to other spring manufacturers. In such cases it is necessary for the spring manufacturer and his engineer to ascertain by experiment just what any grade of wire will do. In this way an exact average of the physical properties of the wire is obtained.

Wire springs are likely to be very small; for instance, the writer has one particular spring in mind which was so small that 38,000 were required to weigh one pound. For measuring wire of any kind, it is better to use a micrometer caliper, thus recording the dimensions in decimal parts of an inch, instead of attempting to use gages of any kind.

Spring Ends

Extension springs are furnished with an infinite number of different kinds of ends, many of which have become so commonly known in the trade that they are regarded as standard. The two hooks at opposite ends of a spring may be made in line and in the same or different planes; and they may be located at the center of the spring or on one side. A further variation may be obtained by arranging the ends out of line and they may even be located at right angles to each other. Some common forms of spring ends are illustrated in Fig. 22, and in referring to these illustrations it will be well to remember that the direction in which the springs should be wound ought always to be specified when giving an order. It must also be borne in mind that tables and calculations of the form presented in this article apply only to the flexible part of the spring. Ends of various kinds, such as hooks, plugs, etc., are really not part of the spring proper, such ends being merely mechanical appliances for attaching the spring to other machine elements.

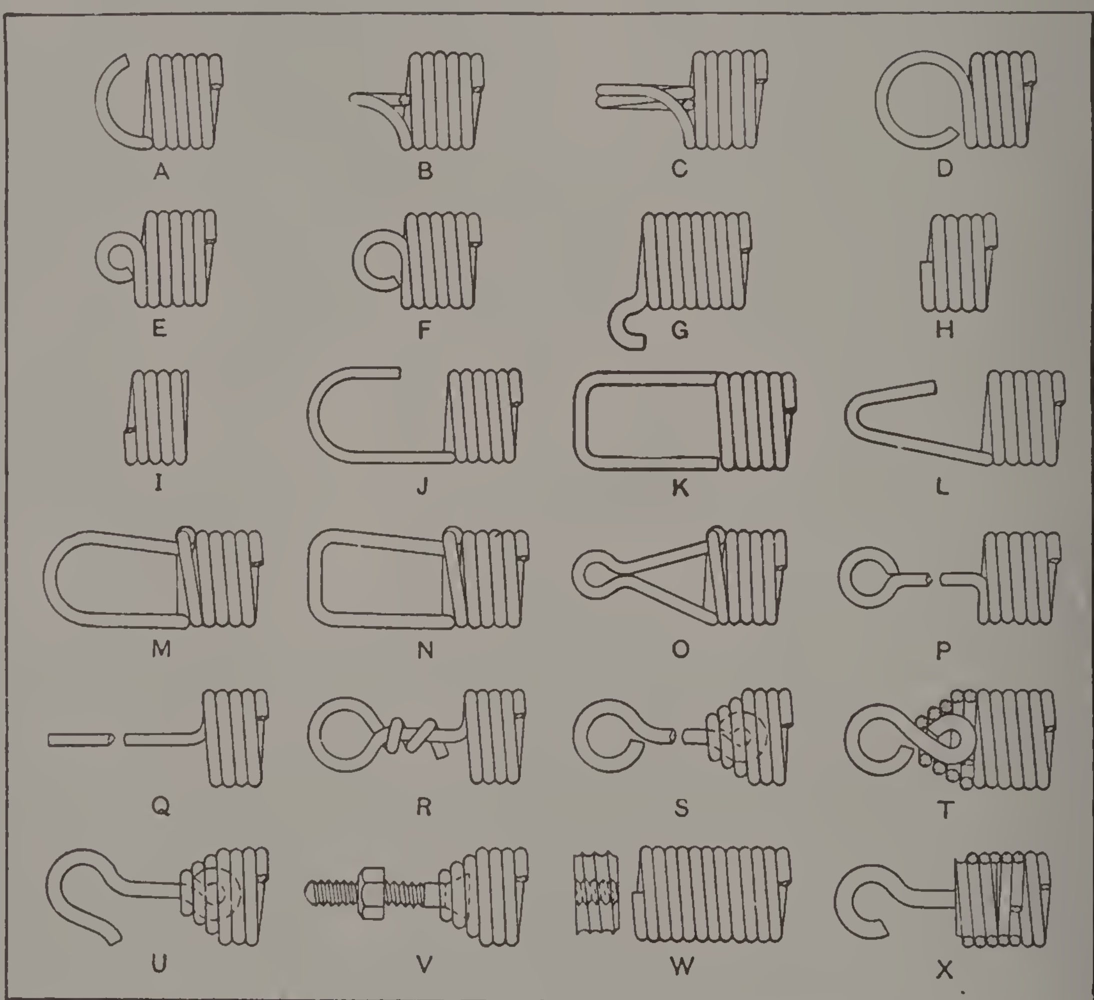


Fig. 22. The Spring A has a Regular Machine Hook over Center; B, Regular Hand Loop over Center; C, Double-coil Hand Loop over Center; D, Regular Hand Loop at Side; E, Small Eye at one Side; F, Small Eye over Center; G, Small Hook at one Side; H, Plain End; I, Ground End; J, Long Hook; K, Long Square Hook; L, V-hook; M, Loop knotted or secured to Spring; N, Square Loop knotted or secured to Spring; O, Knotted Eye; P, Extended Eye; Q, Straight End (usually annealed so that it can be twisted); R, Annealed End, eyed and twisted; S, Tapered End with Extended Swivel Eye; T, Tapered End with Regular Swivel Eye; U, Tapered End with Swivel Hook; V, Tapered End with Swivel Bolt; W, Plain End with Plug to screw into Place; and X, Plain End with Hooked Plug.

WIRE SPRING TABLE

Weight per inch of solid height equals $A \times d^2$. Capacity of coil equals $B \times d^2$, where d is the diameter of bar in inches.

$\frac{D}{d}$.	Length per Inch of Solid Height.	Weight per Inch of Solid Height. A.	Free Height per Inch of Solid Height.	Capacity. B.
3	9.4248	2.0973	1.1795	10,472
$3\frac{1}{16}$	9.6212	2.1410	1.1871	10,258
$3\frac{1}{8}$	9.8175	2.1847	1.1948	10,053
$3\frac{3}{16}$	10.0138	2.2284	1.2027	9,856
$3\frac{1}{4}$	10.2102	2.2721	1.2107	9,666
$3\frac{5}{16}$	10.4066	2.3158	1.2189	9,484
$3\frac{3}{8}$	10.6029	2.3595	1.2272	9,308
$3\frac{7}{16}$	10.7992	2.4031	1.2357	9,139
$3\frac{1}{2}$	10.9956	2.4468	1.2443	8,976
$3\frac{9}{16}$	11.1920	2.4905	1.2531	8,819
$3\frac{5}{8}$	11.3883	2.5342	1.2621	8,666
$3\frac{11}{16}$	11.5846	2.5779	1.2712	8,520
$3\frac{3}{4}$	11.7810	2.6216	1.2805	8,378
$3\frac{13}{16}$	11.9774	2.6653	1.2899	8,240
$3\frac{7}{8}$	12.1737	2.7090	1.2995	8,107
$3\frac{15}{16}$	12.3700	2.7527	1.3092	7,979
4	12.5664	2.7964	1.3191	7,854
$4\frac{1}{16}$	12.7628	2.8401	1.3292	7,733
$4\frac{1}{8}$	12 9591	2.8838	1.3394	7,616
$4\frac{3}{16}$	13.1554	2.9275	1.3498	7,502
$4\frac{1}{4}$	13.3518	2.9712	1.3603	7,392
$4\frac{5}{16}$	13.5482	3.0149	1.3709	7,285
$4\frac{3}{8}$	13.7445	3.0586	1.3818	7,181
$4\frac{7}{16}$	13.9408	3.1022	1.3928	7,080
$4\frac{1}{2}$	14.1372	3.1459	1.4039	6,981
$4\frac{9}{16}$	14.3336	3.1896	1.4152	6,886
$4\frac{5}{8}$	14.5299	3.2333	1.4267	6,793
$4\frac{11}{16}$	14.7262	3.2770	1.4383	6,702
$4\frac{3}{4}$	14.9226	3.3207	1.4500	6,614
$4\frac{13}{16}$	15.1190	3.3644	1.4620	6,528
$4\frac{7}{8}$	15.3153	3.4081	1.4740	6,444
$4\frac{15}{16}$	15.5116	3.4518	1.4863	6,363
5	15.7080	3.4955	1.4987	6,283
$5\frac{1}{16}$	15.9044	3.5392	1.5112	6,206
$5\frac{1}{8}$	16.1007	3.5829	1.5239	6,130
$5\frac{3}{16}$	16.2970	3.6266	1.5367	6,056
$5\frac{1}{4}$	16.4934	3.6703	1.5498	5,984
$5\frac{5}{16}$	16.6898	3.7140	1.5629	5,916
$5\frac{3}{8}$	16.8861	3.7576	1.5763	5,845
$5\frac{7}{16}$	17.0824	3.8013	1.5897	5,748

WIRE SPRING TABLE

$\frac{D}{d}$.	Length per Inch of Solid Height.	Weight per Inch of Solid Height. *A*.	Free Height per Inch of Solid Height.	Capacity. *B*.
$5\frac{1}{2}$	17.2788	3.8450	1.6034	5712
$5\frac{9}{16}$	17.4752	3.8887	1.6171	5648
$5\frac{5}{8}$	17.6715	3.9324	1.6311	5585
$5\frac{11}{16}$	17.8678	3.9761	1.6452	5524
$5\frac{3}{4}$	18.0642	4.0198	1.6595	5464
$5\frac{13}{16}$	18.2606	4.0635	1.6739	5405
$5\frac{7}{8}$	18.4569	4.1072	1.6884	5347
$5\frac{15}{16}$	18.6532	4.1509	1.7032	5291
6	18.8496	4.1946	1.7187	5236
$6\frac{1}{16}$	19.0460	4.2383	1.7331	5182
6	19.2423	4.2820	1.7483	5129
$6\frac{3}{16}$	19.4386	4.3257	1.7636	5077
$6\frac{1}{4}$	19.6350	4.3694	1.7791	5027
$6\frac{5}{16}$	19.8314	4.4131	1.7948	4977
$6\frac{3}{8}$	20.0277	4.4567	1.8106	4928
$6\frac{7}{16}$	20.2240	4.5004	1.8266	4880
$6\frac{1}{2}$	20.4204	4.5441	1.8427	4833
$6\frac{9}{16}$	20.6168	4.5878	1.8590	4787
$6\frac{5}{8}$	20.8131	4.6315	1.8754	4742
$6\frac{11}{16}$	21.0094	4.6752	1.8920	4698
$6\frac{3}{4}$	21.2058	4.7189	1.9088	4654
$6\frac{13}{16}$	21.4022	4.7626	1.9257	4612
$6\frac{7}{8}$	21.5985	4.8063	1.9428	4570
$6\frac{15}{16}$	21.7948	4.8500	1.9600	4528
7	21.9912	4.8937	1.9774	4488
$7\frac{1}{16}$	22.1876	4.9374	1.9949	4448
$7\frac{1}{8}$	22.3839	4.9811	2.0126	4409
$7\frac{3}{16}$	22.5802	5.0248	2.0304	4371
$7\frac{1}{4}$	22.7766	5.0685	2.0484	4333
$7\frac{5}{16}$	22.9730	5.1122	2.0666	4296
$7\frac{3}{8}$	23.1693	5.1558	2.0849	4260
$7\frac{7}{16}$	23.3656	5.1995	2.1033	4224
$7\frac{1}{2}$	23.5620	5.2432	2.1220	4189
$7\frac{9}{16}$	23.7584	5.2869	2.1407	4154
$7\frac{5}{8}$	23.9547	5.3306	2.1597	4120
$7\frac{11}{16}$	24.1510	5.3743	2.1788	4087
$7\frac{3}{4}$	24.3474	5.4180	2.1980	4054
$7\frac{13}{16}$	24.5438	5.4617	2.2174	4021
$7\frac{7}{8}$	24.7401	5.5054	2.2370	3938
$7\frac{15}{16}$	24.9364	5.5491	2.2567	3958
8	25.1328	5.5928	2.2765	3927
$8\frac{1}{16}$	25.3292	5.6365	2.2966	3897
$8\frac{1}{8}$	25.5255	5.6802	2.3167	3867

WIRE SPRING TABLE

$\frac{D}{d}$	Length per Inch of Solid Height.	Weight per Inch of Solid Height. A	Free Height per Inch of Solid Height.	Capacity. B
$8\frac{3}{16}$	25.7218	5.7239	2.3371	3837
$8\frac{1}{4}$	25.9182	5.7676	2.3576	3808
$8\frac{5}{16}$	26.1146	5.8112	2.3782	3779
$8\frac{3}{8}$	26.3109	5.8549	2.3990	3751
$8\frac{7}{16}$	26.5072	5.8986	2.4200	3723
$8\frac{1}{2}$	26.7036	5.9423	2.4411	3696
$8\frac{9}{16}$	26.9000	5.9860	2.4624	3669
$8\frac{5}{8}$	27.0963	6.0297	2.4838	3642
$8\frac{11}{16}$	27.2926	6.0734	2.5054	3616
$8\frac{3}{4}$	27.4890	6.1171	2.5271	3590
$8\frac{13}{16}$	27.6854	6.1608	2.5490	3565
$8\frac{7}{8}$	27.8817	6.2045	2.5711	3540
$8\frac{15}{16}$	28.0780	6.2482	2.5933	3515
9	28.2744	6.2919	2.6156	3491

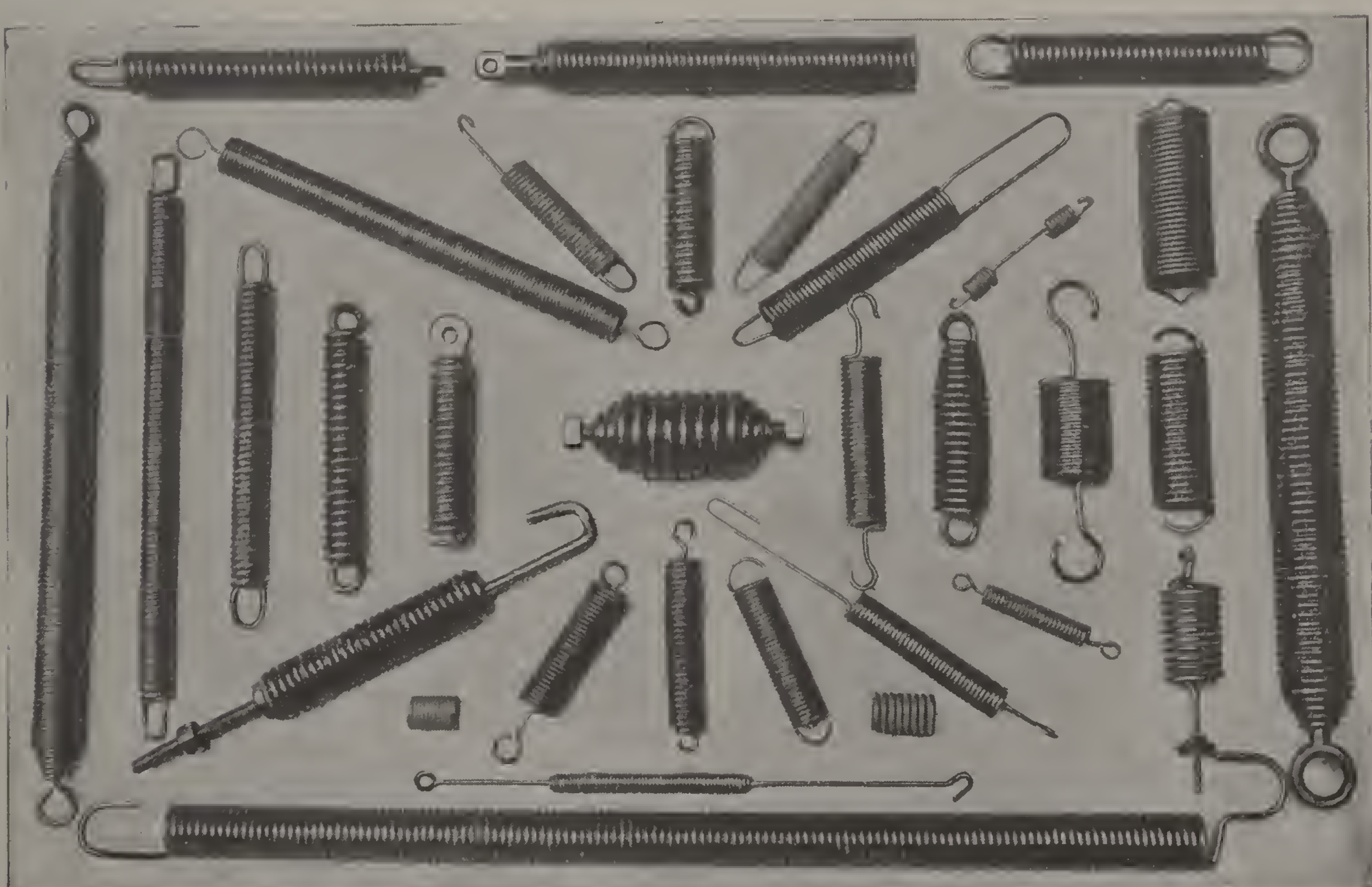


Fig. 23. Group of Wire Springs

CHAPTER VIII.

HELICAL SPRINGS OF CONSTANT DIAMETERS BUT VARIABLE ELLIPTICAL AND RECTANGULAR BARS

We wish in this article to investigate the effect which the variation of one dimension of the bar has upon the spring properties. The outside and inside diameters of the spring will first be kept constant, which will result in keeping D constant and also that dimension of the bar upon which it is not wound, the side of the bar which we see as we look at the end of the spring; while the side of the bar upon which it is wound will be varied. We will express the unchanging bar dimension, or the $\dfrac{O.D. - I.D.}{2}$ by d and the variation of the other side we will express in terms of d as $x\,d$, x being a variable.

It will then be clear that when x equals unity we are dealing with sections such as circles, squares, etc.; when x exceeds unity we are winding the bar upon the flat; when x is less than unity we are winding the bar upon its edge. Thus:

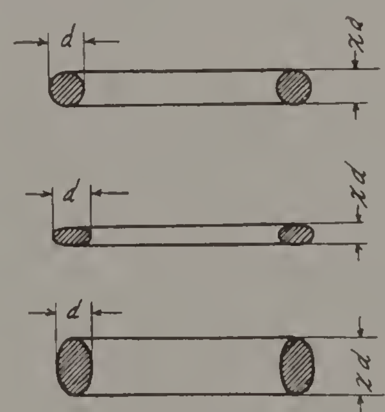


Fig. 24.

Springs made of flat material are useful where either a great amount of compression is required or where the load springs are to carry at some compressed length is such that round or square material cannot be used in their manufacture.

Springs made of flat material coiled on edge are used where springs of considerable strength are required, and where springs are required to compress into a short space.

Springs made of round and square material are sometimes coiled with all the pitch or space between coils which they will stand, and when in use they are often worked very rapidly to almost their closed length. Such springs are not durable but it is possible to construct springs which have the same strength but are made of flat material coiled on edge. Because of the extra number of coils which can be put into this type of spring, the strain on each coil is not as

great as it would be had round or square material been used, and a much more durable spring is the result.

Springs are often required to carry a given load at some specified point of compression, and yet the space left for the spring, *i. e.*, the inside and outside diameters, is such that in order to give the strength required, the springs cannot be made from round or square material. In such cases, springs made of flat material coiled on the flat side are used.

Elliptical Section—Load

The polar moment of an elliptical section whose outside dimensions are a and b is for ordinary sections approximately:

$$J = \pi \left(\frac{(b\,a^3 + a\,b^3)}{64} \right)$$

From which the polar moment of an elliptical section whose outside dimensions are d and $x\,d$ is,

$$J = \frac{\pi\,d^4}{64}\,(x + x^3)$$

Also in an elliptical section the distance from the neutral axis to the remotest fiber will be

$$c = \frac{d}{2} \text{ when } x \text{ equals unity or less than unity.}$$

And

$$c = \frac{x\,d}{2} \text{ when } x \text{ is more than unity.}$$

The twisting moment to which any bar is subject is

$$T = \frac{S\,J}{c}$$

We then have two expressions to derive for this twisting moment, as follows:

1. For x equal *unity or less*, we have

$$T = (S) \left(\frac{2}{d}\right) \left(\frac{\pi\,d^4}{64}\right) (x + x^3)$$

$$= \frac{\pi\,S\,d^3}{32} (x + x^3)$$

But also we know that,

$T = P\,R$ or in the case of a spring of this kind

$$T = \frac{P\,D}{2}$$

$$\therefore \frac{P\,D}{2} = \frac{\pi\,S\,d^3}{32}\,(x + x^3)$$

$$P = \frac{\pi\,S}{16}\,(x + x^3)\,\frac{d^3}{D}$$

Which may be expressed as

$$P = \frac{\pi S}{16} X_1 \frac{d^3}{D}$$

where $X_1 = x + x^3$

Or for steel,

$$P = Y_1 \frac{d^3}{D}$$

$$\text{where } Y_1 = \frac{\pi S}{16} X_1 = 15708 X_1$$

2. In like manner if x is *more* than unity,

$$T = (S)\left(\frac{2}{x\,d}\right)\left(\frac{\pi d^4}{64}\right)(x + x^3)$$

$$= \frac{\pi S d^3}{32}(1 + x^2)$$

$$= \frac{P D}{2}$$

$$P = \frac{\pi S}{16}(1 + x^2)\frac{d^3}{D}$$

Which may be expressed as

$$P = \frac{\pi S}{16} X_1 \frac{d^3}{D}$$

In which case

$$X_1 = (1 + x^2)$$

Or for steel,

$$P = Y_1 \frac{d^3}{D}$$

where $Y_1 = \dfrac{\pi S}{16} X_1 = 15708\ X_1$, as before.

Elliptical Section—Deflection

If a spring is made of an elliptical bar coiled upon the edge xd, as shown, then the length of the bar will be

$$l = \frac{\pi h D}{x\,d}$$

and the value of c will be $\dfrac{d}{2}$ when x is unity or less than unity, and $\dfrac{x\,d}{2}$ when x is more than unity.

Then since $f = \dfrac{S R l}{c G}$

and since $R = \dfrac{D}{2}$

we have,

$$f = S\left(\frac{D}{2}\right)\left(\frac{\pi h D}{x d}\right)\frac{1}{c G}$$

$$f = \frac{\pi S h D^2}{2 x c G d}$$

Or where x is unity or less, substituting c,

$$f = \frac{\pi S h}{x^2 G}\left(\frac{D}{d}\right)^2$$

And when x is more than unity substituting c,

$$f = \frac{\pi S h}{x \ G}\left(\frac{D}{d}\right)^2$$

$$f = \frac{1}{x^2} \quad \frac{\pi S}{G} \cdot \left(\frac{D}{d}\right)^2 h$$

which may be expressed as follows:

$$f = X_2 \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

When

$$X_2 = \frac{1}{x^2}$$

Or for steel

$$f = Y_2 \left(\frac{D}{d}\right)^2 h$$

where

$$Y_2 = \frac{\pi S}{G} X_2 = .019946\, X_2$$

Rectangular Section—Load

The polar moment of a rectangular section whose outside dimensions are a and b is,

$$J = \frac{ab^3 + a^3 b}{12}$$

From which the polar moment of a rectangular section whose outside dimensions are d and xd, is

$$J = \frac{d^4}{12}(x + x^3)$$

In a rectangular section the distance from the neutral axis to the remotest fibre will be, always

$$c = \frac{\sqrt{d^2 + x^2 d^2}}{2} = \frac{d}{2} \qquad \sqrt{1 + x^2}$$

Then since

$$T = \frac{S\,J}{c}$$

$$T = \frac{S\,d^3}{6}\;\frac{(x + x^3)}{\sqrt{1 + x^2}} = \frac{P\,D}{2}$$

$$\therefore\quad P = \left(\frac{S}{3}\right)\left(x\sqrt{1 + x^2}\right)\frac{d^3}{D}$$

which may be expressed as

$$P = \frac{S}{3}\,X_3\,\frac{d^3}{D}$$

Where

$$X_3 = x\sqrt{1 + x^2}$$

Or for steel

$$P = Y_3\,\frac{d^3}{D}$$

Where

$$Y_3 = \frac{S}{3}\,X_3 = 26666\;X_3$$

Rectangular Section—Deflection

If a spring is made of a rectangular section and coiled upon the edge xd, as shown, then the length of the bar will be

$$l = \frac{\pi\,h\,D}{x\,d}$$

and the value of c will be $\sqrt{d^2 + x^2\,d^2}$ or $d\sqrt{1 + x^2}$

Then since $f = \dfrac{S\,R\,l}{c\,G}$ and since $R = \dfrac{D}{2}$

we have

$$f = S\left(\frac{D}{2}\right)\left(\frac{\pi\,h\,D}{x\,d}\right)\left(\frac{1}{d\,G\sqrt{1 + x^2}}\right)$$

$$f = \frac{1}{x\sqrt{1 + x^2}}\left(\frac{\pi\,S}{G}\right)\left(\frac{D}{d}\right)^2 h$$

which may be expressed as

$$f = \frac{\pi\,S}{G}\left(X_4\right)\left(\frac{D}{d}\right)^2 h$$

where $X_4 = \dfrac{1}{x\sqrt{1 + x^2}}$

Or for steel

$$f = Y_4\left(\frac{D}{d}\right)^2 h \qquad \text{where } Y_4 = \frac{\pi\,S}{G}\,X_4 = .019946\;X_4$$

Table of Loads and Deflections for Various Values of x

From the foregoing we have the following table of co-efficients for steel springs, where

O. D. = a constant
I. D. = a constant
d = a constant

and where the edge upon which the bar is rolled is equal to xd, x being a variable.

	Load $= P = Y\left(\frac{d^3}{D}\right)$		Deflection $= f = Y\left(\frac{D}{d}\right)^2 h$	
x	Y_1 — Elliptical Sections	Y_3 — Rectangular Sections	Y_2 — Elliptical Sections	Y_4 — Rectangular Sections
.1	1,587	2,680	.19946	.198470
.2	3,267	5,439	.099730	.097794
.3	5,137	8,352	.066487	.063683
.4	7,289	11,488	.049865	.046299
.5	9,818	14,907	.039892	.035681
.6	12,818	18,659	.033242	.028506
.7	16,383	22,786	.028494	.023343
.8	20,609	27,320	.024933	.019469
.9	25,588	32,289	.022162	.016473
1.0	31,416	37,712	.019946	.014104
1.1	34,715	43,608	.016484	.012197
1.2	38,328	49,986	.013851	.010641
1.3	42,255	45,858	.011802	.0093549
1.4	46,496	46,232	.010177	.0082810
1.5	51,051	72,112	.0088649	.0073760
1.6	55,920	80,504	.0077914	.0066071
1.7	61,104	89,413	.0069017	.0059488
1.8	66,602	98,839	.0061562	.0053813
1.9	72,414	108,787	.0055252	.0048894
2.0	78,540	119,259	.0049865	.0044601
3.0	157,080	252,986	.0022162	.0021025
4.0	267,036	439,804	.0012466	.0012094
5.0	408,408	679,878	.00079784	.00078235
6.0	581,196	973,254	.00055406	.00054652
7.0	785,400	1,319,950	.00040706	.00040297
8.0	1,121,020	1,719,970	.00031166	.00030925
9.0	1,288,056	2,173,321	.00024625	.00024474
10.0	1,586,508	2,680,001	.00019946	.00019847

SPRINGS MADE OF FLAT MATERIAL

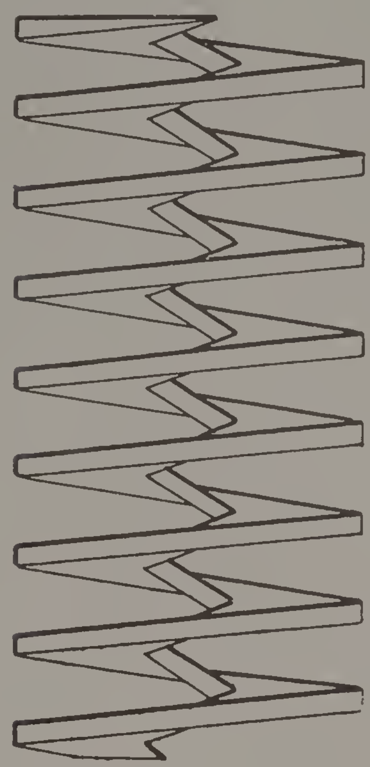


Fig. 25.
Front view of R. H. spring, wound on edge.

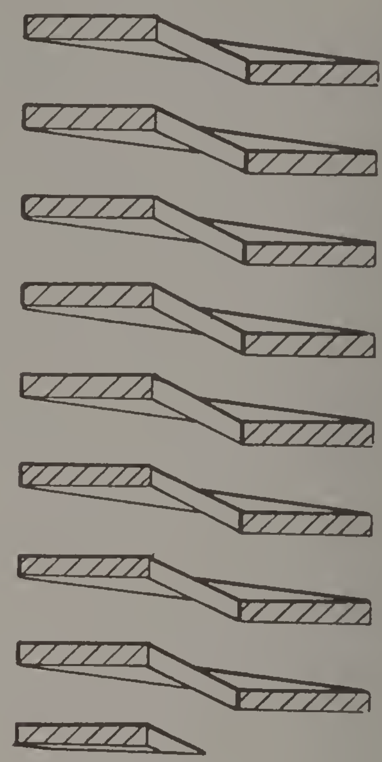


Fig. 26.
Sectional view of R. H. spring, wound on edge.

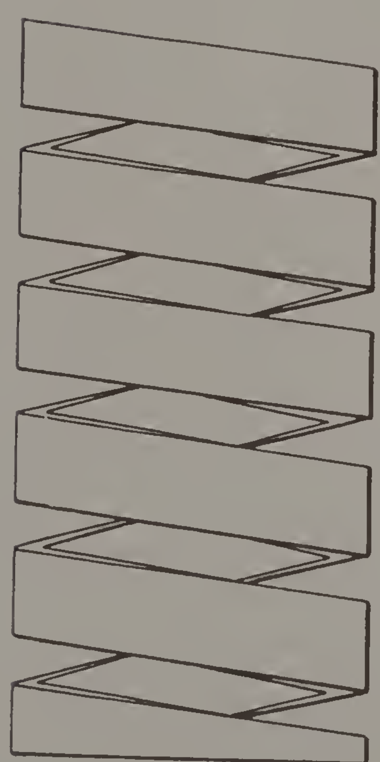


Fig. 27.
Front view of L. H. Spring, wound on flat.

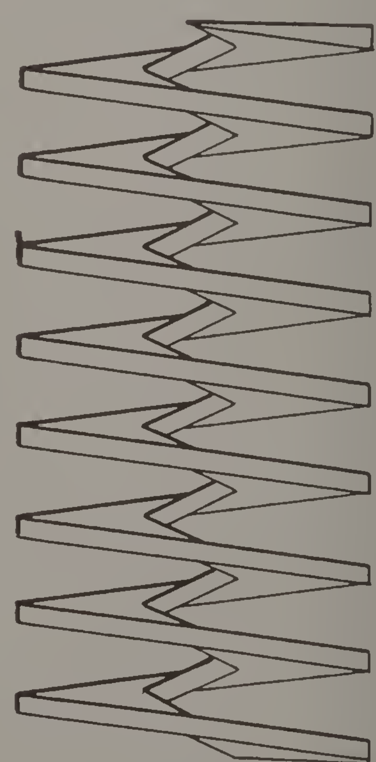


Fig. 28.
Showing spring wound L. H. and on edge.

MATHEMATICAL TABLES—FOURTH POWERS MEAN DIAMETERS

No.	Fourth Power	No.	Fourth Power	No.	Fourth Power	No.	Fourth Power
1/16	0.000015	3 7/16	139.62745	6 13/16	2153.90261	10 3/16	10771.35887
1/8	0.000024	3 1/2	150.06250	6 7/8	2234.03929	10 1/4	11038.12865
3/16	0.000124	3 9/16	161.07176	6 15/16	2316.39164	10 5/16	11309.82394
1/4	0.003906	3 5/8	172.67603	7	2401.00000	10 3/8	11586.50431
5/16	0.009537	3 11/16	184.89625	7 1/16	2487.90532	10 7/16	11868.22971
3/8	0.019776	3 3/4	197.75391	7 1/8	2577.14869	10 1/2	12155.06250
7/16	0.036636	3 13/16	211.27077	7 3/16	2668.77172	10 9/16	12444.75546
1/2	0.062500	3 7/8	225.46899	7 1/4	2762.81644	10 5/8	12744.29331
9/16	0.100113	3 15/16	240.37111	7 5/16	2859.32497	10 11/16	13046.81479
5/8	0.152588	4	256.00000	7 3/8	2958.34009	10 3/4	13355.79595
11/16	0.223404	4 1/16	272.37892	7 7/16	3059.90479	10 13/16	13667.98502
3/4	0.316406	4 1/8	289.53149	7 1/2	3164.06250	10 7/8	13986.75836
13/16	0.435806	4 3/16	307.48171	7 9/16	3270.85695	10 15/16	14311.02750
7/8	0.586182	4 1/4	326.25391	7 5/8	3380.33231	11	14641.00000
15/16	0.772477	4 5/16	345.87282	7 11/16	3492.58296	11 1/16	14976.59724
1	1.000000	4 3/8	366.36352	7 3/4	3607.50394	11 1/8	15317.93017
1 1/16	1.274429	4 7/16	387.75148	7 13/16	3725.29031	11 3/16	15665.06417
1 1/8	1.601806	4 1/2	410.06250	7 7/8	3845.93777	11 1/4	16018.06612
1 3/16	1.988541	4 9/16	433.32277	7 15/16	3969.49224	11 5/16	16377.00104
1 1/4	2.441406	4 5/8	457.55884	8	4096.00000	11 3/8	16741.93430
1 5/16	2.967544	4 11/16	482.79762	8 1/16	4225.50779	11 7/16	17112.93161
1 3/8	3.574462	4 3/4	509.06641	8 1/8	4358.06272	11 1/2	17490.06250
1 7/16	4.270035	4 13/16	536.39284	8 3/16	4493.71216	11 9/16	17873.39228
1 1/2	5.062500	4 7/8	564.80493	8 1/4	4632.50395	11 5/8	18262.98892
1 9/16	5.960464	4 15/16	594.33107	8 5/16	4774.48612	11 11/16	18658.91961
1 5/8	6.972901	5	625.00000	8 3/8	4919.70724	11 3/4	19061.25420
1 11/16	8.109146	5 1/16	656.84084	8 7/16	5068.21632	11 13/16	19470.05944
1 3/4	9.378906	5 1/8	689.88306	8 1/2	5220.06250	11 7/8	19885.40594
1 13/16	10.792252	5 3/16	724.15651	8 9/16	5375.29544	11 15/16	20307.36352
1 7/8	12.359618	5 1/4	759.69141	8 5/8	5533.96508	12	20736.00000
1 15/16	14.091811	5 5/16	796.51832	8 11/16	5782.99683	12 1/16	21171.38711
2	16.000000	5 3/8	834.66821	8 3/4	5861.81645	12 1/8	21613.59335
2 1/16	18.095719	5 7/16	874.17238	8 13/16	6031.09990	12 3/16	22062.69244
2 1/8	20.390869	5 1/2	915.06250	8 7/8	6204.02366	12 1/4	22518.75360
2 3/16	22.897720	5 9/16	957.37062	8 15/16	6380.63971	12 5/16	22981.84886
2 1/4	25.628906	5 5/8	1001.12917	9	6561.00000	12 3/8	23452.05060
2 5/16	28.597427	5 11/16	1046.37089	9 1/16	6745.15722	12 7/16	23929.30971
2 3/8	31.816650	5 3/4	1093.12893	9 1/8	6933.16432	12 1/2	24414.06250
2 7/16	35.300309	5 13/16	1141.43678	9 3/16	7125.07447	12 9/16	24906.00804
2 1/2	39.062500	5 7/8	1191.32839	9 1/4	7320.94145	12 5/8	25405.37082
2 9/16	43.117691	5 15/16	1242.83792	9 5/16	7520.81914	12 11/16	25912.19509
2 5/8	47.480714	6	1296.00000	9 3/8	7724.76197	12 3/4	26426.56672
2 11/16	52.166764	6 1/16	1350.84964	9 7/16	7932.82473	12 13/16	26948.55687
2 3/4	57.191406	6 1/8	1407.42210	9 1/2	8145.06250	12 7/8	27478.24215
2 13/16	62.570571	6 3/16	1465.75316	9 9/16	8361.53080	12 15/16	28015.69826
2 7/8	68.061807	6 1/4	1525.87894	9 5/8	8582.28544	13	28561.00000
2 15/16	74.458023	6 5/16	1587.83571	9 11/16	8807.38258	13 1/16	29114.22381
3	81.000000	6 3/8	1651.66037	9 3/4	9036.87895	13 1/8	29675.44519
3 1/16	87.985319	6 7/16	1717.39013	9 13/16	9270.83136	13 3/16	30244.74264
3 1/8	95.367431	6 1/2	1785.06250	9 7/8	9509.29715	13 1/4	30822.19107
3 3/16	103.228775	6 9/16	1854.71534	9 15/16	9752.33397	13 5/16	31407.86974
3 1/4	111.566406	6 5/8	1926.38696	10	10000.00000	13 3/8	32001.85559
3 5/16	120.39919	6 11/16	2000.11596	10 1/16	10252.35321	13 7/16	32604.22728
3 3/8	129.74634	6 3/4	2075.94137	10 1/8	10509.45334	13 1/2	33215.06250

MATHEMATICAL TABLES—CUBES OF MEAN DIAMETERS

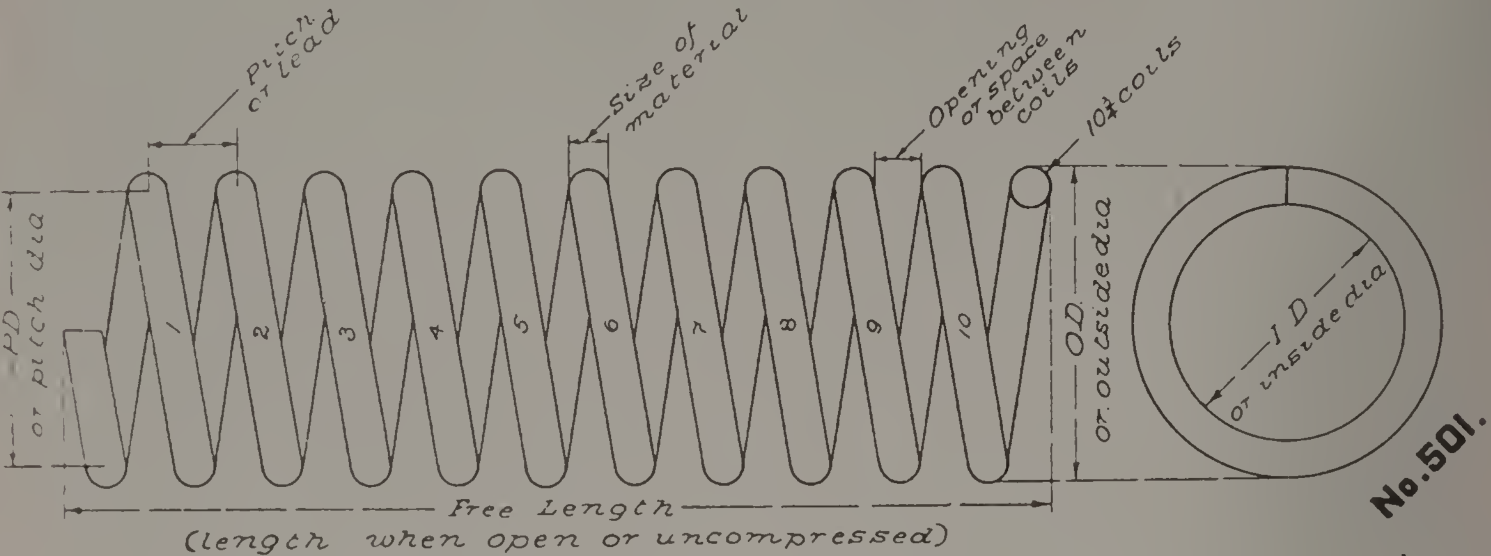
No.	Cube	No.	Cube	No.	Cube	No.	Cube
1/16	0.000244	3 7/16	40.618896	6 13/16	316.169189	10 3/16	1057.311279
1/8	0.001953	3 1/2	42.875000	6 7/8	324.951172	10 1/4	1076.890625
3/16	0.006592	3 9/16	45.213135	6 15/16	333.894287	10 5/16	1096.710205
1/4	0.015625	3 5/8	47.634766	7	343.000000	10 3/8	1116.771484
5/16	0.030518	3 11/16	50.141357	7 1/16	352.269775	10 7/16	1137.075928
3/8	0.052734	3 3/4	52.734375	7 1/8	361.705078	10 1/2	1157.625000
7/16	0.083740	3 13/16	55.415283	7 3/16	371.307373	10 9/16	1178.201660
1/2	0.125000	3 7/8	58.185547	7 1/4	381.078125	10 5/8	1199.462891
9/16	0.177979	3 15/16	61.046631	7 5/16	391.018799	10 11/16	1220.754639
5/8	0.244141	4	64.000000	7 3/8	401.130859	10 3/4	1242.306641
11/16	0.324951	4 1/16	67.047119	7 7/16	411.415771	10 13/16	1264.091064
3/4	0.421875	4 1/8	70.189453	7 1/2	421.875000	10 7/8	1286.138672
13/16	0.536377	4 3/16	73.428467	7 9/16	432.510010	10 15/16	1308.426768
7/8	0.669922	4 1/4	76.765625	7 5/8	443.322266	11	1331.000000
15/16	0.823975	4 5/16	80.202393	7 11/16	454.313232	11 1/16	1353.816650
1	1.000000	4 3/8	83.740234	7 3/4	465.484375	11 1/8	1376.892578
1 1/16	1.199463	4 7/16	87.380615	7 13/16	476.837158	11 3/16	1400.229248
1 1/8	1.423828	4 1/2	91.125000	7 7/8	488.373047	11 1/4	1423.828125
1 3/16	1.674561	4 9/16	94.974854	7 15/16	500.093506	11 5/16	1447.690673
1 1/4	1.953125	4 5/8	98.931641	8	512.000000	11 3/8	1471.818359
1 5/16	2.260986	4 11/16	102.996826	8 1/16	524.093994	11 7/16	1496.212646
1 3/8	2.599609	4 3/4	107.171875	8 1/8	536.376953	11 1/2	1520.875000
1 7/16	2.970459	4 13/16	111.458252	8 3/16	548.850342	11 9/16	1545.806885
1 1/2	3.375000	4 7/8	115.857422	8 1/4	561.515625	11 5/8	1571.009766
1 9/16	3.814697	4 15/16	120.370850	8 5/16	574.374268	11 11/16	1596.485107
1 5/8	4.291016	5	125.000000	8 3/8	587.427734	11 3/4	1622.234375
1 11/16	4.805420	5 1/16	129.746338	8 7/16	600.677490	11 13/16	1648.259033
1 3/4	5.359375	5 1/8	134.611328	8 1/2	614.125000	11 7/8	1674.560547
1 13/16	5.954346	5 3/16	139.596436	8 9/16	627.771729	11 15/16	1701.140381
1 7/8	6.591796	5 1/4	144.703125	8 5/8	641.619141	12	1728.000000
1 15/16	7.273193	5 5/16	149.932861	8 11/16	655.668701	12 1/16	1755.140869
2	8.000000	5 3/8	155.287109	8 3/4	669.921875	12 1/8	1782.564453
2 1/16	8.773682	5 7/16	160.767334	8 13/16	684.380127	12 3/16	1810.272217
2 1/8	9.595703	5 1/2	166.375000	8 7/8	699.044922	12 1/4	1838.265625
2 3/16	10.467529	5 9/16	172.111572	8 15/16	713.917725	12 5/16	1866.546143
2 1/4	11.390625	5 5/8	177.978516	9	729.000000	12 3/8	1895.115234
2 5/16	12.366455	5 11/16	183.977295	9 1/16	744.293213	12 7/16	1923.964600
2 3/8	13.396484	5 3/4	190.109375	9 1/8	759.798828	12 1/2	1953.125000
2 7/16	14.482178	5 13/16	196.376221	9 3/16	775.518311	12 9/16	1982.568604
2 1/2	15.625000	5 7/8	202.779297	9 1/4	791.453125	12 5/8	2012.306641
2 9/16	16.826416	5 15/16	209.320068	9 5/16	807.604736	12 11/16	2042.340576
2 5/8	18.087891	6	216.000000	9 3/8	823.974609	12 3/4	2072.671875
2 11/16	19.410889	6 1/16	222.820557	9 7/16	840.564209	12 13/16	2103.302002
2 3/4	20.796875	6 1/8	229.783203	9 1/2	857.375000	12 7/8	2134.232422
2 13/16	22.247314	6 3/16	236.889404	9 9/16	874.408447	12 15/16	2165.464600
2 7/8	23.763672	6 1/4	244.140625	9 5/8	890.666016	13	2197.000000
2 15/16	25.347412	6 5/16	251.538330	9 11/16	919.149170	13 1/16	2228.840088
3	27.000000	6 3/8	259.083984	9 3/4	926.859375	13 1/8	2260.986328
3 1/16	28.722900	6 7/16	266.779053	9 13/16	944.798096	13 3/16	2293.440180
3 1/8	30.517578	6 1/2	274.625000	9 7/8	962.966797	13 1/4	2326.203125
3 3/16	32.385498	6 9/16	282 623291	9 15/16	981.366943	13 5/16	2359.276611
3 1/4	34.328125	6 5/8	290.775391	10	1000.000000	13 3/8	2392.662109
3 5/16	36.346924	6 11/16	299.082764	10 1/16	1018.867432	13 7/16	2426.361084
3 3/8	38.443359	6 3/4	307.546875	10 1/8	1037.970703	13 1/2	2460.375000

MATHEMATICAL TABLE—FIFTH POWERS OF BAR DIAMETERS

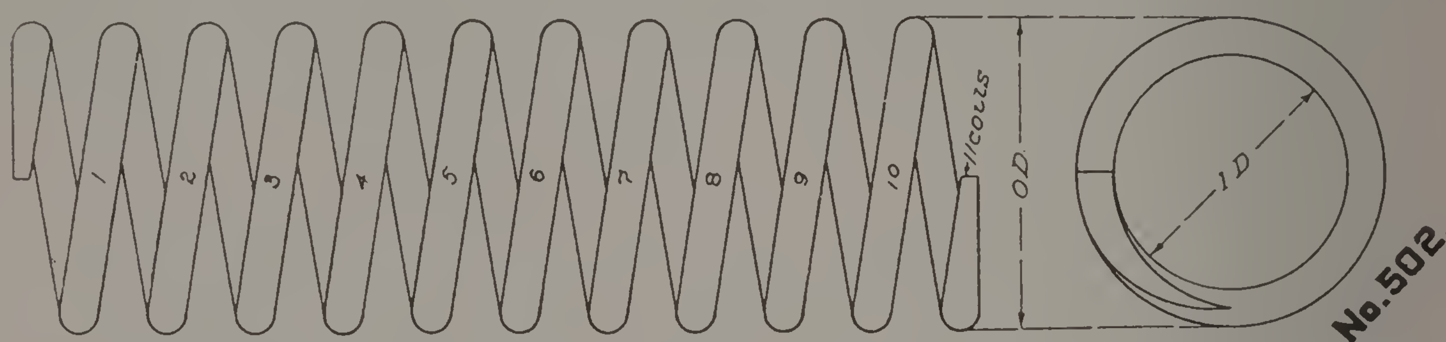
No.	Fifth Power	No.	Fifth Power	No.	Fifth Power	No.	Fifth Power
$\frac{1}{16}$	0.000000953674	$\frac{9}{16}$	0.056313	$1\frac{1}{16}$	1.35408	$1\frac{9}{16}$	9.3132
$\frac{1}{8}$	0.0000305176	$\frac{5}{8}$	0.095367	$1\frac{1}{8}$	1.80203	$1\frac{5}{8}$	11.3310
$\frac{3}{16}$	0.000231743	$\frac{11}{16}$	0.153590	$1\frac{3}{16}$	2.36139	$1\frac{11}{16}$	13.6842
$\frac{1}{4}$	0.000976562	$\frac{3}{4}$	0.237305	$1\frac{1}{4}$	3.05176	$1\frac{3}{4}$	16.4131
$\frac{5}{16}$	0.00298023	$\frac{13}{16}$	0.354093	$1\frac{5}{16}$	3.89490	$1\frac{13}{16}$	19 5610
$\frac{3}{8}$	0.00741577	$\frac{7}{8}$	0.512909	$1\frac{3}{8}$	4.91489	$1\frac{7}{8}$	23.1743
$\frac{7}{16}$	0 0160284	$\frac{15}{16}$	0.724196	$1\frac{7}{16}$	6.13818	$1\frac{15}{16}$	27.3029
$\frac{1}{2}$	0.0312500	1	1.000000	$1\frac{1}{2}$	7.59375	2	32.0000

COMPRESSION SPRINGS.

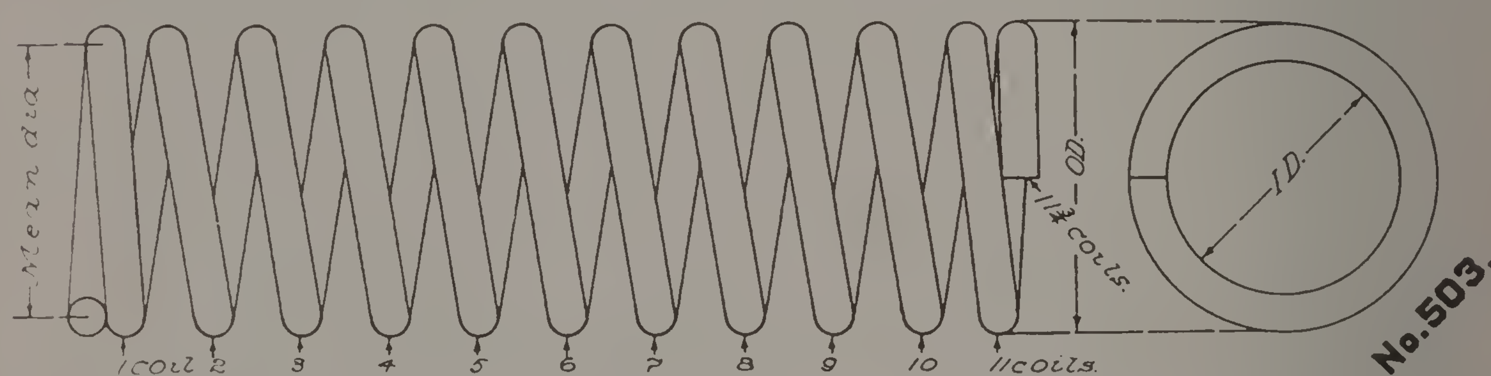
MADE OF ROUND MATERIAL



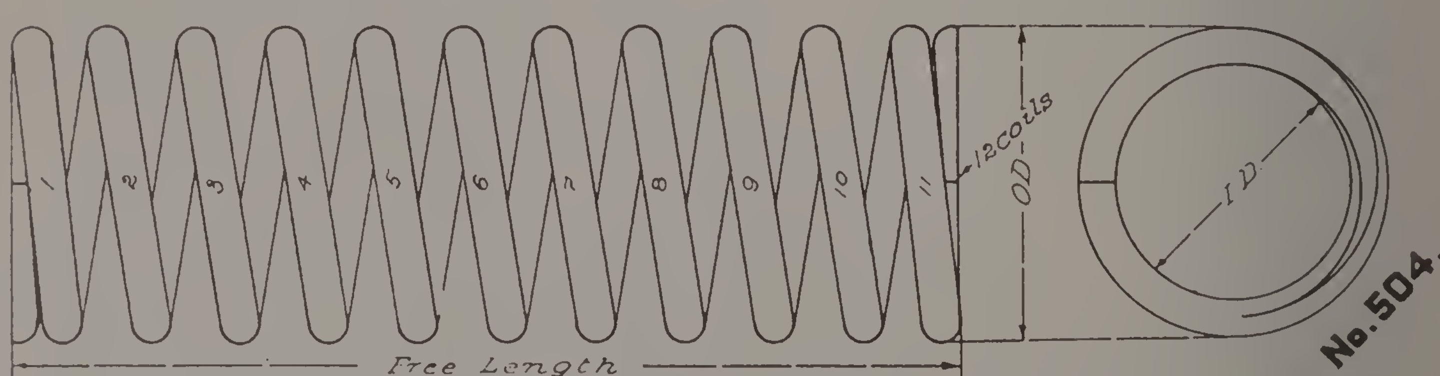
Above drawing shows a plain end spring, coiled right hand, with 10¾ coils



Above drawing shows a spring with plain ends ground; coiled left hand, made with 11 coils.



Above drawing shows a spring with squared or closed ends, not ground, coiled right hand, made with 11¾ coils. (Approximately 10 active or working coils.)

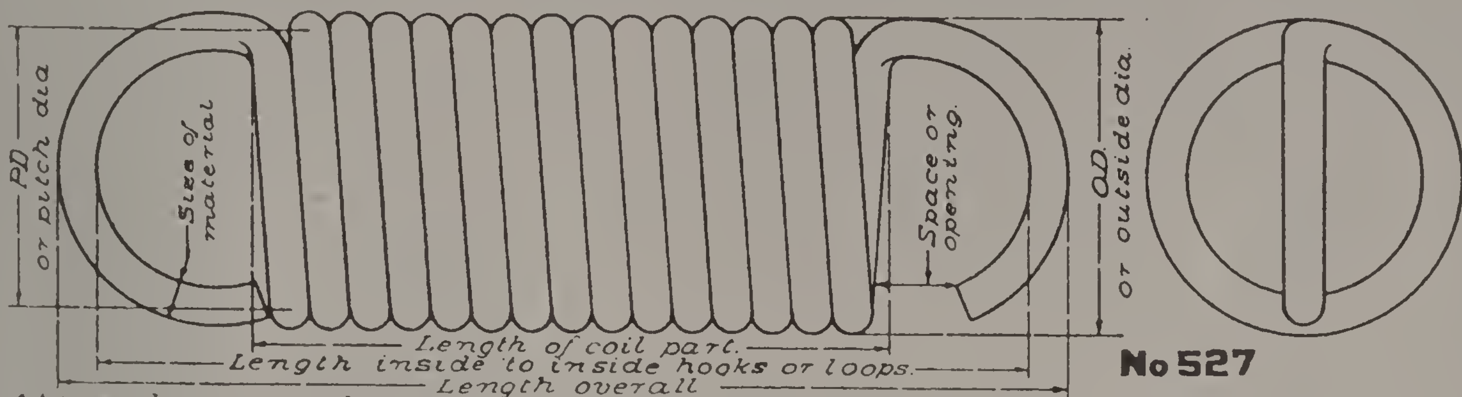


Above drawing shows a spring with squared or closed ends, ground, coiled right hand, made with 12 coils (Approximately 10½ active coils)

The above drawings represent the usual method of showing compression springs made of round steel with the various finishes of ends. They also show our method of counting the coils in a spring and it will be seen that we count the total number of coils in the spring regardless of the finish of ends. In the case of springs made with squared ends, not ground and also springs made with squared ends ground, the working coils (sometimes called active or effective coils) are less than the total coils.

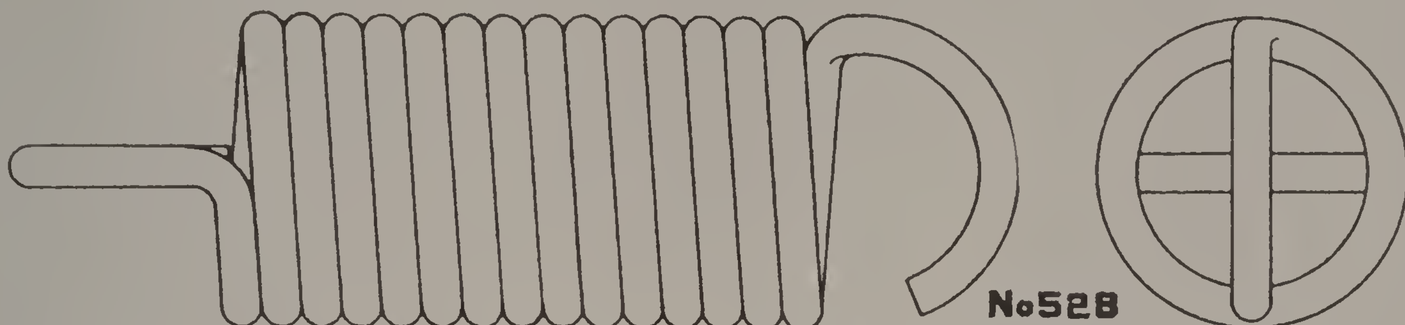
EXTENSION SPRINGS.

MADE OF ROUND MATERIAL.



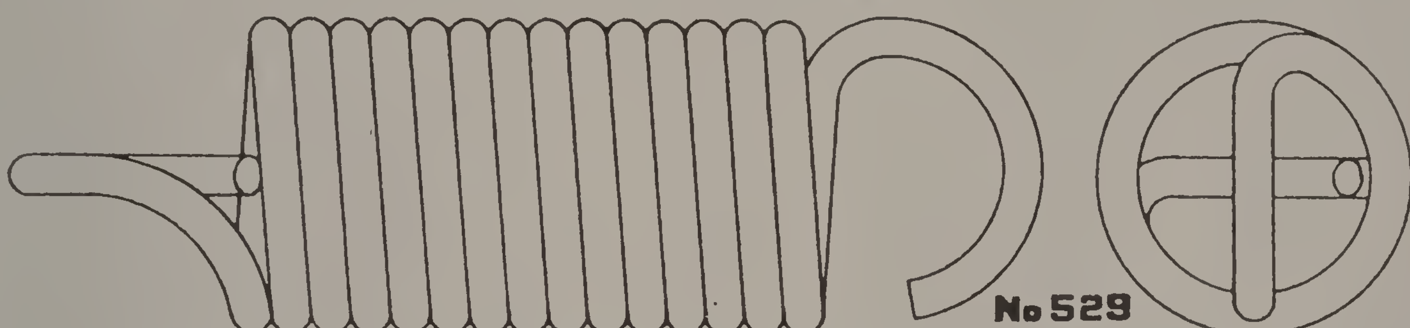
No 527

Above drawing shows an extension spring with regular machine loop and hook, in line or in same plane



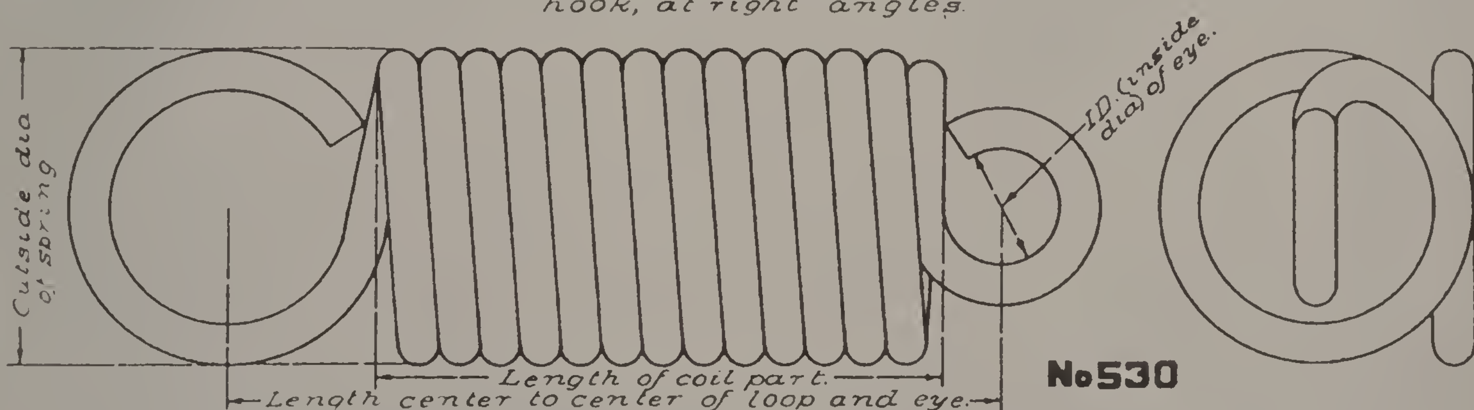
No 528

Above drawing shows an extension spring with regular machine loop and hook, at right angles.



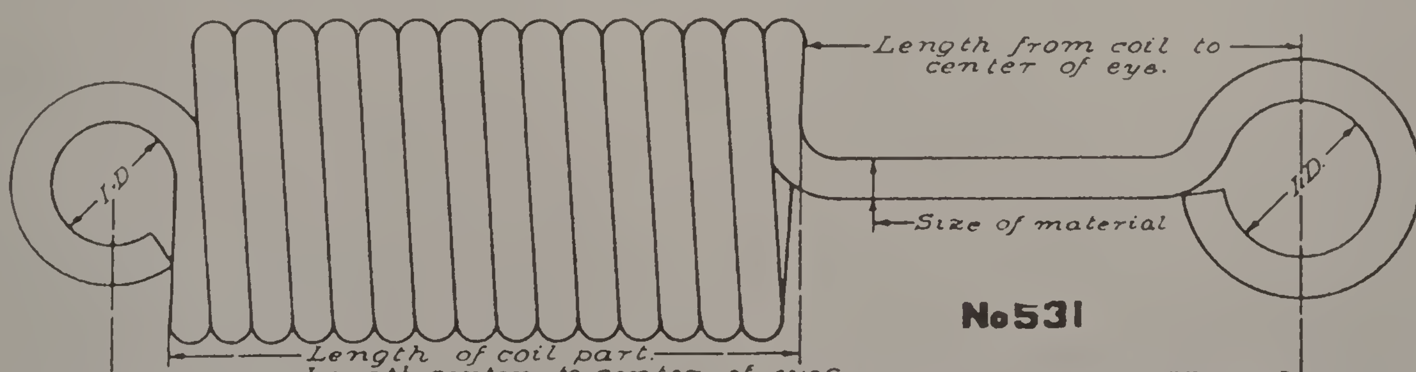
No 529

Above drawing shows an extension spring with regular hand loop and hook, at right angles.



No 530

Above drawing shows an extension spring with regular loop of one coil on side and special crossover eye



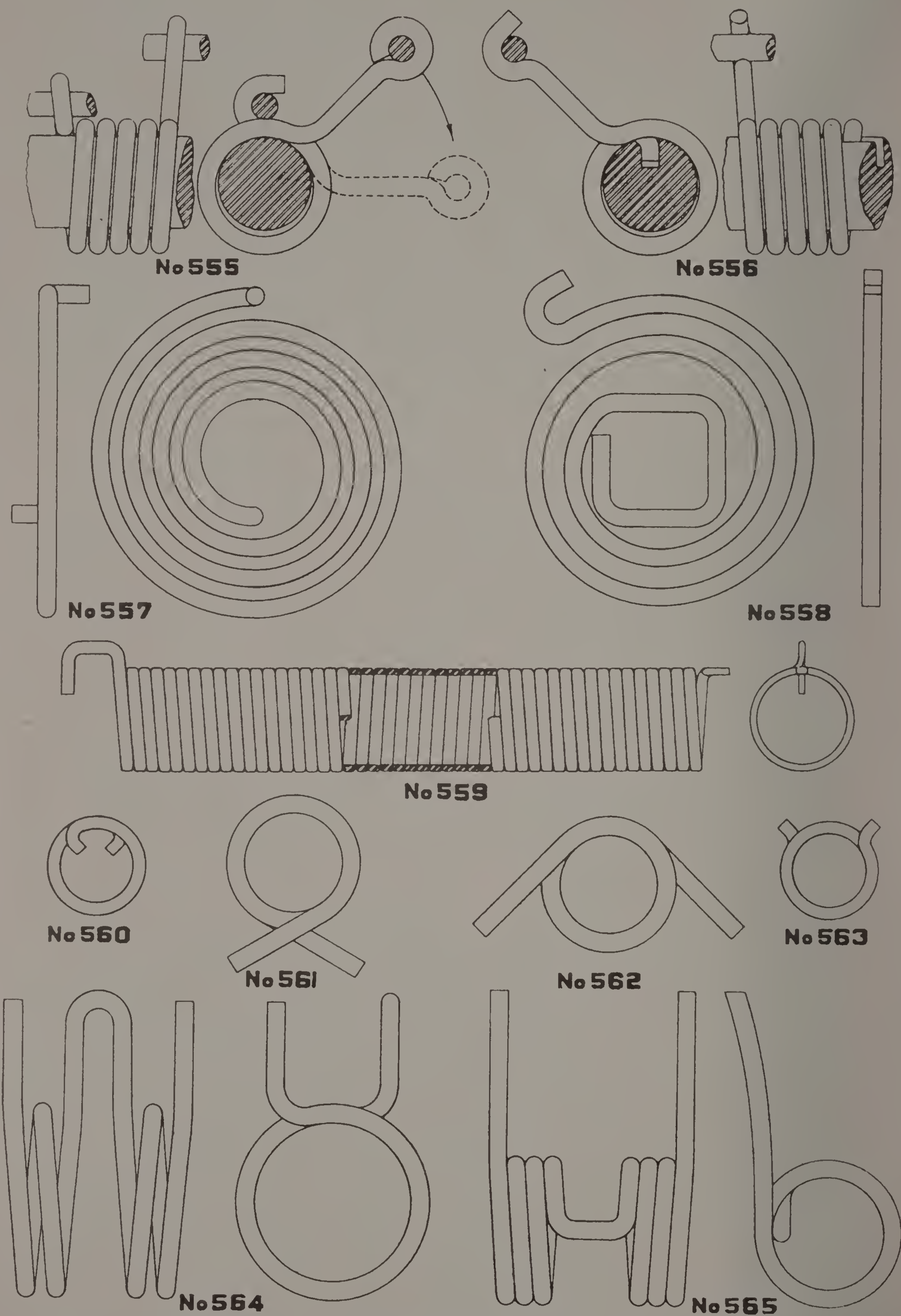
No 531

Above drawing shows an extension spring with special crossover eye and an extended eye from center of coil, eyes in above spring in line.
(Can be furnished at right angle also)

The above drawings represent the usual method of showing extension springs, with a few types of hook or loop ends, such as are commonly used. Extension springs can be furnished with any style of ends, to meet special conditions.

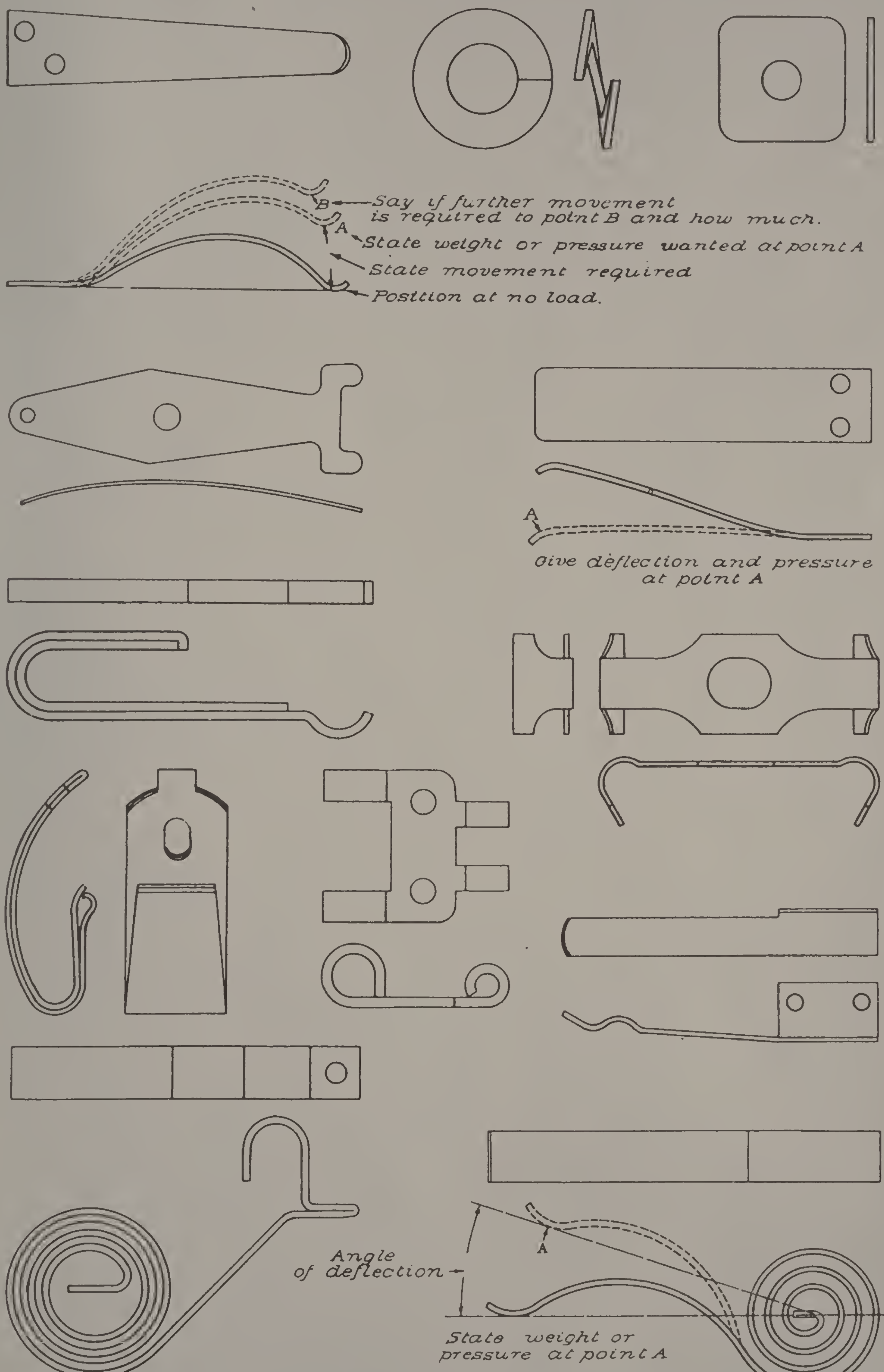
TORSION SPRINGS.

MADE OF ROUND SQUARE AND FLAT MATERIAL



The above drawings represent only a few of the unlimited shapes in which torsion springs can be made.
We can furnish any size and shape

FLAT SPRINGS.



Above is merely a suggestion for drawings required on orders for flat springs. Furnish full size drawings with all dimensions.